NICHOLSONS INTERMEDIATE ARITHMETIC

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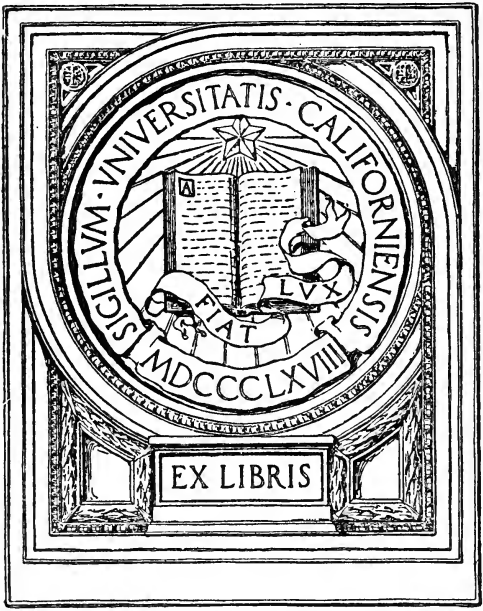

INTERMEDIATE

ARITHMETIC

ON THE INDUCTIVE METHOD, WITH PARALLEL MENTAL AND WRITTEN EXERCISES

BY

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PRACTICAL EDUCATIONAL SERIES.

Chambers' Twenty Lessons in Book-keeping.
Hansell's Penmanship.
Nicholson's Primary Arithmetic.
Nicholson's Intermediate Arithmetic.
Nicholson's Complete Arithmetic.

PREFACE.

The chief difference between a good and an inferior Arithmetic is not so much a question of matter and rules, as it is of method in the presentation and development of principles. In the former, few mathematicians would be bold enough to lay claim to originality; but in the latter every one will, perhaps, admit there is room for improvement.

In the preparation of this three-book series, consisting of a Primary, an Intermediate, and a Complete Arithmetic, the author has been influenced by the following considerations:

1°. Arithmetic treats of the *whole* and its *parts*. These are the magnitudes or objects about which Analysis and Synthesis are conversant, and on the consideration of which depends the solution of every problem. Hence, the early introduction of these terms, and frequent reference to them in the deduction of succeeding principles, are of the greatest importance.

2°. By Induction a pupil is led by easy steps, by familiar illustrations and commonplace parallelisms, into a clear apprehension of principles and definitions. Hence, each subject should be introduced with inductive exercises.

3°. Pupils advance intelligently in any new subject just in proportion as they perceive in it a continuation of the principles with which they are familiar. Hence, whatever of sameness and of difference there is in the old and the new should be made as conspicuous as possible.

4°. Mental and written work are equally important, and should be mutually supplemental. A problem intended for written work should, in general, be preceded by a parallel question designed for mental, and also as an inductive exercise.

5°. The representing of objects by the first letters of their names, as, **a** for apple, and **b** for boy or box, is not only a matter of convenience, but serves to lead pupils into the habit of generalization.

6°. Pictures assist the child to some extent in the conception of combining and resolving numbers by counting, adding, subtracting, etc., but are not so useful in this particular as objects themselves. Hence, the introduction of object exercises is a prominent feature of the first two books of the series.

On the whole, the series is not the product of preconceived opinions as to what should constitute matter and method, but the embodiment of the results of many years experience in teaching mathematics.

The present treatise is intended primarily to prepare pupils for the Complete Arithmetic; secondly, to meet the wants of those who desire only a knowledge of those practical applications of numbers which are most frequently used in ordinary business transactions. It is divided into two parts.

The First Part is devoted to a few lessons in primary arithmetic, embodying the more important features of the "Grube Method," with such additions as to bring it into conformity with the principles already enunciated. This part may be omitted by those who have completed the Primary Arithmetic, at the discretion of the teacher.

The Second Part embraces a very thorough elementary course in Notation and Numeration, Addition, Subtraction, Multiplication, Division, Divisors and Multiples, Common and Decimal Fractions, United States Money, Compound Numbers, some Important Practical Applications, and Percentage, including Commission, Profit and Loss and Interest. Special attention is invited to the simple, progressive, and practical treatment of these subjects, especially Division, Fractions, Decimals and Interest.

The Author acknowledges his indebtedness to many writers upon this subject, both of this and of other countries, whose able treatises have been consulted with pleasure and profit.

J. W. N.

Baton Rouge, La., *June*, 1885.

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PART II.

PART I.

INTERMEDIATE ARITHMETIC.

LESSON I.

COUNTING TEN.

First Row	. . .	**a**
Second "	. . .	**a a**
Third "	. . .	**a a a**
Fourth "	. . .	**a a a a**
Fifth "	. . .	**a a a a a**
Sixth "	. . .	**a a a a a a**
Seventh "	. . .	**a a a a a a a**
Eighth "	. . .	**a a a a a a a a**
Ninth "	. . .	**a a a a a a a a a**
Tenth "	. . .	**a a a a a a a a a a**

DIAGRAM OF A'S.

Count the **a**'s in each row, beginning at the top.

Count the **a**'s in each row, beginning at the bottom.

Which is the first row? The second? The third? The fourth? etc.

How many **a**'s are in the first row? The third row? The ninth? The fifth? The second? The seventh? The fourth? The eighth? The sixth? The tenth?

Figure 0 stands for none,	Figure 5 stands for five,
" 1 " " one,	" 6 " " six,
" 2 " " two,	" 7 " " seven,
" 3 " " three,	" 8 " " eight,
" 4 " " four,	" 9 " " nine,

10 stands for ten.

Which row has two **a**'s in it? 5 **a**'s? 9 **a**'s? 7 **a**'s?

Make 6 taps on your slate. Make three marks. Get up and make 8 steps. Hold up 10 fingers; 5 fingers; 3 fingers.

Make all the figures on your slate. Thus:

0 1 2 3 4 5 6 7 8 9 10

Count 10 axes. *Ans.* 1 ax, 2 axes, 3 axes, 4 axes, 5 axes, 6 axes, 7 axes, 8 axes, 9 axes, 10 axes.

Count 10 arms, 10 apples.

What may **a** stand for? *Ans.* An ax, or an arm, or an apple.

Count 10 **a**'s. *Ans.* 1 **a**, 2 **a**, 3 **a**, 4 **a**, 5 **a**, 6 **a**, 7 **a**, 8 **a**, 9 **a**, 10 **a**.

Count 8 boys; 6 bats; 7 birds.

What stands for a boy, or a bat, or a bird, etc.? *Ans.* **b**.

Count 9 **b**'s.

What stands for a cat, or a cow, or a cap, etc.? *Ans.* **c**.

Count 5 **c**'s.

What stands for a horse, or an hour, or a hen?

What stands for a finger, or a fan, or a fob?

What is a Unit? *Ans.* One of the things counted.

What is the unit of 8 fans? 9 boys? 7 girls? 5 **a**'s? *Ans.* 1 **a**.

Suggestions to Teachers.—I. This first lesson should be recited with each of the following until it is well learned. Copy the diagram of *a*'s on the board for recitation, occasionally substituting other letters for *a*. In this, as in subsequent lessons, ask such additional questions as will enlist the interest and meet the wants of the individual pupil.

II. It is important that the pupil should obtain a clear idea of the number of units represented by a figure. Hence, it is recommended that the teacher write on the board some figure, as 7, without calling its name, and require the pupil to make that many marks, or steps, or hold up that many fingers Pupils should practice writing the figures until they can make them readily and correctly.

III. The sign + should always be read *and*, and — *less*, and not plus and minus, until the student becomes perfectly familiar with their primary meaning.

IV. In illustrating principles and processes, use material objects as far as possible.

Note. — Prior to the subject of Fractions, the term parts denote integers, and the two complemental parts of a number are denoted by the ***two parts.***

Lesson II.

About the Number Two.

What lesson is this? What is it about? Hold up two fingers. How many hands have you? Write 2 on your slate. Write 2 two times. How many horses are in the picture? Do 2 horses make a pair?

Is 1 horse the whole or a part of the pair? What are the **two parts** of 2 horses? *Ans.* 1 horse and 1 horse. What are the **two parts** of 2 men? 2 cows? 2 dollars? 2 houses? 2 **a**'s? 2 **b**'s?

What are the **two parts** of 2? *Ans.* 1 and 1.

ADDITION.—*Uniting the parts.*

How many are 1 horse and 1 horse? 1 cow and 1 cow? 1 mule and 1 mule? 1 apple and 1 apple? 1 **b** and 1 **b**? 1 and 1?

What sign stands for and? *Ans.* +. What is it called? *Ans.* And. Go to the board and make it. What sign stands for are and equals? *Ans.* =. Make it on the board. Write this: 1+1=2 on the board. How is it read? *Ans.* 1 and 1 are 2.

SUBTRACTION.—*Taking away one part.*

Two horses are together; if 1 horse is taken away, how many horses will be left? 1 horse taken from 2 horses leaves how many? Jane had 2 roses, but gave 1 rose to Mary; how many roses did Jane have left? How many are 2 roses less 1 rose? 2 peaches less 1 peach? 2 pins less 1 pin? 2 birds less 1 bird?

What sign stands for less? *Ans.* —. Make it on the board. What is it called? *Ans.* Less. Write this: 2—1=1 on the board. How is it read? *Ans.* 2 less 1 is 1, or 1 from 2 leaves 1.

MULTIPLICATION.—*Uniting equal parts.*

Make 1 mark. Make 1 mark 2 times. How many are 2 times 1 mark? 2 times 1 horse? Hold up 2 fingers. Now hold up 2 times 1 finger. Frank has 2

dimes, and Charles has 2 times 1 dime; which has the more money?

What sign stands for times? *Ans.* $\times$. Make it on the board. Write this: $2 \times 1 = 2$ on the board. How is it read? *Ans.* 2 times 1 are 2.

Division. — *Measuring by a part.*

Does 2 contain its parts? What are its parts? Does 2 contain 1 and 1? Does it contain 1 2 times?

How many times do 2 horses contain 1 horse? How many times can you take 1 pint from 2 pints? How many times, then, do 2 pints contain 1 pint?

What sign stands for contains? *Ans.* $\div$. Make it on the board. Write this: $2 \div 1 = 2$ on the board. How is it read? *Ans.* 2 contains 1, 2 times, or 2 divided by 1 equals 2.

Lesson III.

About the Number Three.

What lesson is this? What is it about? Hold up 3 fingers. Tap your slate 3 times. Make 3 on the board. Make 3 three times. Is the wagon loaded with 3 bales? Are 3 bales the whole load or a part of the load? Are 2 bales the whole load or a part

of the load? Is 1 bale a part of the load? Do 2 bales and 1 bale make 3 bales? Are 1 bale and 2 bales the same as 2 bales and 1 bale? What are the **two parts** of 3 bales? *Ans.* 1 bale and 2 bales. What are the **two parts** of 3 barrels? 3 birds? 3 dollars? 3 **d**'s? 3 **m**'s?

What are the **two parts** of 3? *Ans.* 1 and 2, or 2 and 1.

ADDITION.—*Uniting the parts.*

How many are 2 bales and 1 bale when put together? 2 horses and 1 horse? 1 dime and 2 dimes? 2 **d**'s and 1 **d**? 1 **c** and 2 **c**'s? 1 and 2? What are the two parts of 3? When you put them together, do they make 3? Ann has 2 plums and Susan has 1 plum; how many plums have they together? Ben has one marble and Ike has 2 marbles; if they put them in a sack how many will be in the sack?

Write these, $2+1=3$ and $1+2=3$, and read them.

SUBTRACTION.—*Taking away one part.*

How many bales would be left on the wagon if 1 bale were rolled off? If 2 bales were rolled off? How many are left when 1 is taken from 3? When 2 is taken from 3? What are the **two parts** of 3? When one of them is taken from 3, is the other left? Jane had 3 roses, but gave 1 rose to Mary; how many roses did Jane have left? Ann had 3 cherries but gave 2 cherries to Ben; how many did Ann then have? How many are 3 roses less 1 rose? 3 cherries less 2 cherries? 3 **r**'s less 2 **r**'s? 3 **c**'s less 1 **c**? 3 less 1?

Copy and read, $3-1=2$; $3-2=1$; $3-3=0$.

James killed 1 bird and John killed 3 birds; how many more did John kill than James?

MULTIPLICATION.—*Uniting equal parts.*

How many are 1 and 1 and 1? How many times is 1 taken? 3 times 1 are how many? How many are 3 times 1 bale? 3 times 1 horse? 3 times 1 **h**? 3 times 1 **a**? Julia has 1 rose, and Mary has 3 times as many as Julia; how many roses has Mary? Show me 3 fingers. Now show me 3 times 1 finger.

Copy and read, $3 \times 1 = 3$.

DIVISION.—*Measuring by a part.*

Does the load,—3 bales, contain its parts? Does it contain 1 bale and 1 bale and 1 bale? Does it contain 1 bale 3 times? Put 3 books on the table. Now take off 1 book at a time until all are removed. How many times did you take off 1 book? How many times, then, do 3 books contain 1 book? If a boy carries off a bushel of corn at a time, how many trips will he have to make to carry off 3 bushels? How many times, then, do 3 bushels contain 1 bushel?

Copy and read, $3 \div 1 = 3$; $3 \div 3 = 1$.

Learn and recite:

THE TABLE OF THREE.

0 and 3 are 3	0 from 3 leaves 3
1 and 2 are 3	1 from 3 leaves 2
2 and 1 are 3	2 from 3 leaves 1
3 and 0 are 3	3 from 3 leaves 0
3 times 1 are 3	1 in 3 3 times

NOTE.—0 is called *none*; thus, *none* and 3 are 3.

Lesson IV.

About the Number Four.

What lesson is this? What is it about? Is 4 more than 3? How many more? Show me 4 books. In the

picture is a class of girls; how many girls are there? Are 3 girls the whole, or a part of the class? Are 2 girls the whole, or a part of the class? Is 1 girl a part of the class? If the class were divided into **two parts,** how many girls would be in each part? *Ans.* 1 girl in one part and 3 girls in the other part; or 2 girls in one and 2 girls in the other.

Show me, with 4 fingers, how this would be. What, then, are the **two parts** of 4 girls? *Ans.* 1 girl and 3 girls, and 2 girls and 2 girls.

What are the **two parts** of 4? *Ans.* 1 and 3, 2 and 2.

Addition.—*Uniting the parts.*

How many are 1 girl and 3 girls? 3 girls and 1 girl? 2 girls and 2 girls? 3 horses and 1 horse? 2 mules and 2 mules? 1 **m** and 3 **m**'s? 3 **n**'s and 1 **n**? 2 and 2?

What are the **two parts** of 4? When you put them together, do they make 4? Ann has 1 plum and Emma has 3 plums; how many plums have they together? Ben has 2 marbles and Jake has 2 marbles;

how many have they together? Julia has 3 dolls and Mary has 1 doll more than Julia; how many dolls has Mary?

Copy and read: $1+3=4$; $2+2=4$; $3+1=4$.

SUBTRACTION.—*Taking away one part.*

How many girls would be left in the class, if 1 girl were taken away? If 2 girls were taken away? If 3 girls? If 4 girls? 1 from 4 leaves how many? 2 from 4 leaves how many? 3 from 4? 4 from 4?

What are the **two parts** of 4? When one of them is taken from 4, is the other left? Jane had 4 roses but gave her sister 1 rose; how many roses did Jane then have? Frank had 4 dimes but gave 2 dimes to an old blind man, how many dimes did Frank then have? 4 birds were on a limb, but 3 birds have flown; how many birds are left?

Copy and read: $4-1=3$; $4-2=2$; $4-3=1$.

MULTIPLICATION.—*Uniting equal parts.*

How many are 1, 1, 1 and 1? How many times is the part 1 taken? How many, then, are 4 times 1? How many are 4 times 1 girl? 4 times 1 book? 4 times 1 horse?

How many are 2 and 2? How many times is the part 2 taken? How many, then, are two times 2? How many are 2 times 2 girls? 2 times 2 birds? 2 times 2 roses? Show me 4 fingers. Now show me 4 times 1 finger. Now show me 2 times 2 fingers. Ben has 4 times 1 toy and Frank has 2 times 2 toys; which has the more?

Copy and read: $4\times1=4$; $2\times2=4$.

Division.—*Measuring by a part.*

Does 4 contain its parts? Does it contain 1, 1, 1, and 1? How many times does it contain 1? How many times can 1 gill be taken from 4 gills? How many times, then, do 4 gills contain 1 gill? Does 4 contain the parts 2 and 2? How many times, then, does it contain 2? Are 4 gills the same as 2 gills and 2 gills? How many times, then, do 4 gills contain 2 gills? How many boxes shall I need so that I may put 4 marbles in them and have 2 marbles in each box?

Copy and read: $4 \div 1 = 4$; $4 \div 2 = 2$; $4 \div 4 = 1$.

Object Exercises.

Take 4 blocks, put them on a table, and call them birds. Now divide or separate the birds into **two parts,** every way you can, calling the results thus: 1 bird and 3 birds are 4 birds, etc.

Now take off a bird at a time, calling the results thus: 1 bird from 4 birds leaves 3 birds: 2 birds from 4 birds leave 2 birds, etc. Now put the birds back, one at a time, calling the results thus: 1 time 1 bird is 1 bird; 2 times 1 bird are 2 birds, etc. Next, take the birds off, one at a time, calling the results thus: 1 bird contains 1 bird 1 time; 2 birds contain 1 bird 2 times, etc.

Next, put the birds back, two at a time, calling the results thus: 1 time 2 birds is 2 birds; 2 times 2 birds are 4 birds.

Now take the birds off, two at a time, calling the results thus: 2 birds contain 2 birds 1 time; 4 birds contain 2 birds 2 times.

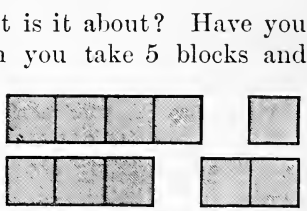
The Table of Four.

0 and	4 are 4	4 from 4	leaves 0
1 and	3 are 4	3 from 4	leaves 1
2 and	2 are 4	2 from 4	leaves 2
3 and	1 are 4	1 from 4	leaves 3
4 times	1 are 4	1 in 4,	4 times
2 times	2 are 4	2 in 4,	2 times

Lesson V.

About the Number Five.

What lesson is this? What is it about? Have you 5 fingers on one hand? Can you take 5 blocks and separate them as they are here pictured? Take 5 rocks and separate them as the blocks are. Which are the first **two parts** of 5 blocks? The second? The third? The fourth? Are 2 blocks and 3 blocks the same as 3 blocks and 2 blocks? Are 1 block and 4 blocks the same as 4 blocks and 1 block?



What are the **two parts** of 5? *Ans.* 1 and 4, 2 and 3.

Addition.—*Uniting the parts.*

Count 5. How many are 1 block and 4 blocks? 2 horses and 3 horses? 3 men and 2 men? 1 cow and 4 cows?

What are the **two parts** of 5? Do they make 5 when united? How many is 1 more than 4? Two more than 3?

Some boys are in a class; if 3 boys are a part of the class, and 2 boys are the other part, how many boys are in the class? If 4 mules and 1 mule are the two parts of a team, how many are in the team? How many are 3 girls and 2 girls? 4 girls and 1 girl? 2 apples and 3 apples? 1 **a** and 4 **a**'s?

Copy and read: $4 + 1 = 5$; $3 + 2 = 5$; $2 + 3 = 5$; $1 + 4 = 5$.

SUBTRACTION.—*Taking away one part.*

Count 5 backward. 5 blocks are together; if 1 block is removed, how many blocks will be left? How many are 1 block from 5 blocks? 2 horses from 5 horses? 3 men from 5 men? 4 cows from 5 cows?

What are the **two parts** of 5? When one of them is taken from 5, is the other left? How many are 1 less than 5? 2 less than 5? 3 less than 5? etc.

If 3 chicks are one part of a brood of 5 chicks, how many chicks are in the other part? 2 cats are a part of 5 cats; what is the other part?

Copy and read: $5 - 1 = 4$; $5 - 2 = 3$; $5 - 3 = 2$; $5 - 4 = 1$; $5 - 5 = 0$.

MULTIPLICATION.—*Uniting equal parts.*

How many are 1, 1, 1, 1, and 1? How many times is the part 1 taken? How many, then, are 5 times 1? How many are 5 times 1 block? Hold up 5 fingers. Now hold up 5 times 1 finger. Thomas has 5 dollars, and Henry has 5 times 1 dollar; which boy has the

more? Are 2 cents and 3 cents the same as 5 times 1 cent?

What does 0 stand for? Are 5 nothings any more than 1 nothing? How many, then, are 5 times 0?

Copy and read: $5 \times 1 = 5$; $5 \times 0 = 0$.

DIVISION.—*Measuring by a part.*

Does 5 contain its parts? Is 5 formed of 5 ones? How many times, then, does it contain 1? Make 5 **a**'s. Now erase 1 **a** at a time until all are gone; how many times did you erase? How many times, then, do 5 **a**'s contain 1 **a**?

Five gallons of water are in a tub; if each horse drinks a gallon, how many horses will drink it all? How many times, then, is 1 gallon contained in 5 gallons?

Copy and read: $5 \div 1 = 5$; $5 \div 5 = 1$.

OBJECT EXERCISES.

Take 5 blocks, put them on a table, and call them hats. Now separate the hats into **two parts** every way you can, calling the results thus: 1 hat and 4 hats are 5 hats, etc.

Now take off 1 hat at a time, calling the results thus: 1 hat from 5 hats leaves 4 hats; 2 hats from 5 hats leave 3 hats, etc.

Now put back 1 hat at a time, calling the results thus: 1 time 1 hat is 1 hat; 2 times 1 hat are 2 hats, etc.

Next take off 1 hat at a time, calling the results thus: 1 hat contains 1 hat 1 time; 2 hats contain 1 hat 2 times, etc.

The Table of Five.

0 and 5 are 5	0 from 5 leaves 5
1 and 4 are 5	1 from 5 leaves 4
2 and 3 are 5	2 from 5 leaves 3
3 and 2 are 5	3 from 5 leaves 2
4 and 1 are 5	4 from 5 leaves 1
5 and 0 are 5	5 from 5 leaves 0
5 times 1 are 5	1 in 5, 5 times

Review Questions.

What are the ***two parts*** of 2? 3? 4? 5?

How many are 1 girl and 1 girl? 1 and 1?

How many are 2 boys and 1 boy? 1 boy and 2 boys?

How many are 1 peach and 3 peaches? 2 flies and 2 flies?

How many are 3 men and 2 men? 1 mule and 4 mules?

How many are left when 1 girl is taken from 2 girls? When 2 men are taken from 3 men? When 1 **m** is taken from 3 **m**'s? When 1 bird is taken from 4 birds? When 2 **b**'s are taken from 4 **b**'s? When 3 pins are taken from 4 pins? When 1 book is taken from 5 books? When 2 fingers are taken from 5 fingers? When 3 **f**'s are taken from 5 **f**'s? When 4 guns are taken from 5 guns?

How many are 2 times 1 gun? 2 times 2 guns? 4 times 1 horse? 3 times 1 mule? 5 times 1 cow? How many does 2 lack of being 3 times 1? Does 3 lack of being 4 times 1? Does 3 lack of being 2 times 2? Does 3 lack of being 5 times 1?

How many times do 2 cents contain 1 cent? Do 3 bushels contain 1 bushel? Do 4 books contain 2 books? Do 5 dollars contain 1 dollar? Do 4 pins contain 1 pin 5 times?

James has 1 marble and John has 3 marbles. How many marbles have James and John together? How many more marbles has John than James? How many does John lack of having 4 times as many as James?

Susan has 2 roses and Mary has 3 roses; how many roses

have both girls? How many more roses has Mary than Susan? How many roses does Mary lack of having 2 times as many as Susan? If Mary should give Susan 2 of her roses, how many would each girl then have?

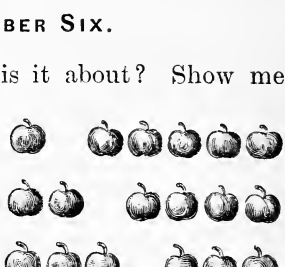
How many are 1+1? 2+2? 3+1? 4+1? 1+3? 3+2? 4+0? 2+3? 2−1? 3−2? 5−3? 5−2? 4−3? 3−1? 4−2? 5−4? 4−4? 5×0? 5×1? 2×2? 4÷1? 4÷2? 3÷1? 2÷1?

NOTE.—The teacher should improvise questions, similar to foregoing, after each lesson. No question, however, should involve a number greater than the subject of the last lesson. The preceding questions may be used with different numbers.

LESSON VI.

ABOUT THE NUMBER SIX.

What lesson is this? What is it about? Show me 6 planks. How many apples in each row of the picture? Is each row separated into **two parts?** How many apples in the **two parts** of the first row? In the **two parts** of the second row? Of the third row? Fourth row? Fifth?



Are the parts of the the first and fifth rows the same? Of the second and fourth? What, then, are the **two parts** of 6 apples? *Ans.* 1 **a** and 5 **a**'s, 2 **a**'s and 4 **a**'s, 3 **a**'s and 3 **a**'s.

What are the **two parts** of 6? *Ans.* { 5 and 1. 4 and 2. 3 and 3.

ADDITION. — *Uniting the parts.*

How many are five apples and 1 apple? 1 apple and 5 apples? 4 **a**'s and 2 **a**'s? 2 **a**'s and 4 **a**'s? 3 **a**'s and 3 **a**'s?

What are the **two parts** of 6? What do they make when united? How many are 4 eggs and 2 eggs? 5 cows and 1 cow? 3 baskets and 3 baskets? 1 orange and 5 oranges?

The two parts of a set of chairs are 2 chairs and 4 chairs; how many chairs in the set? 3 chickens and 3 chickens are the parts of a brood; how many chickens in the brood?

Copy and read: $5+1=6$; $4+2=6$; $3+3=6$; $2+4=6$; $1+5=6$.

SUBTRACTION. — *Taking away one part.*

Six apples are together; how many apples would be left if 1 apple were taken away? If two apples were taken away? If 3 apples? 4 apples? 5 apples? 6 apples?

What are the **two parts** of 6? When one of them is taken from 6, is the other left? A squad of 6 men is divided into **two parts**; if 4 men are in one part, how many are in the other? 6 girls are in one class; part of them are standing, and part are sitting; if 3 are sitting, how many are standing? James had 6 birds in his cage, but 5 birds got out; how many had he left?

Copy and read: $6-1=5$; $6-3=3$; $6-5=1$; $6-2=4$; $6-4=2$; $6-6=0$.

MULTIPLICATION.—*Uniting equal parts.*

How many are 1, 1, 1, 1, 1, and 1? How many times is the part 1 taken? How many, then, are 6 times 1? How many are 2, 2, and 2? How many times is the part 2 taken? How many, then, are 3 times 2? How many are 3 and 3? How many times is 3 taken? How many, then, are 2 times 3? Are 3 times 2 dollars more than 2 times 3 dollars?

John has 2 peaches, and Ben has 3 times as many; how many peaches has Ben? Emma has 3 tulips and Rosa has 2 times as many; how many tulips has Rosa?

Copy and read: $6 \times 1 = 6$; $3 \times 2 = 6$; $2 \times 3 = 6$.

DIVISION.—*Measuring by a part.*

Does 6 contain its parts? Is 6 formed of 6 ones? Of 3 2's? Of 2 3's? How many times, then, does 6 contain 1? 2? 3? Make 6 **a**'s. Now erase 1 **a** at a time. How many times did you erase? How many times, then, is 1 **a** contained in 6 **a**'s?

Make 6 **b**'s. Now erase 2 **b**'s at a time. How many times did you erase? How many times, then, are 2 **b**'s contained in 6 **b**'s?

Make 6 **c**'s. Now erase 3 **c**'s at a time. How many times did you erase? How many times, then, are 3 **c**'s contained in 6 **c**'s?

Copy and read: $6 \div 1 = 6$; $6 \div 2 = 3$; $6 \div 3 = 2$.

OBJECT EXERCISES.

Take 6 blocks, put them on a table, and call them caps.

What stands for cap? *Ans.* **c**.

Now separate the caps into **two parts,** every way you can, calling the results thus: 1 **c** and 5 **c**'s are 6 **c**'s, etc.

Now *take off* 1 cap at a time, calling the result thus: 1 **c** from 6 **c**'s leaves 5 **c**'s; 2 **c**'s from 6 **c**'s leave 4 **c**'s, etc.

Now *put on* 1 cap at a time, calling the results thus: 1 time 1 **c** is 1 **c**; 2 times 1 **c** are 2 **c**'s, etc.

Next *take off* 1 cap at a time, calling the results thus: 1 **c** contains 1 **c** 1 time; 2 **c**'s contain 1 **c** 2 times, etc.

Now *put on* 2 caps at a time, calling the results thus: 1 time 2 **c**'s is 2 **c**'s; 2 times 2 **c**'s are 4 **c**'s, etc.

Next *take off* 2 caps at a time, calling the results thus: 2 **c**'s contain 2 **c**'s 1 time; 4 **c**'s contain 2 **c**'s 2 times, etc.

Now *put on* 3 caps at a time, calling the results thus: 1 time 3 **c**'s is 3 **c**'s; 2 times 3 **c**'s are 6 **c**'s.

Next *take off* 3 caps at a time, calling the results thus: 3 **c**'s contain 3 **c**'s 1 time; 6 **c**'s contain 3 **c**'s 2 times.

Learn and recite—

The Table of Six.

0 and 6 are 6	0 from 6 leaves 6
1 and 5 are 6	1 from 6 leaves 5
2 and 4 are 6	2 from 6 leaves 4
3 and 3 are 6	3 from 6 leaves 3
4 and 2 are 6	4 from 6 leaves 2
5 and 1 are 6	5 from 6 leaves 1
6 and 0 are 6	6 from 6 leaves 0
1 time 6 is 6	1 in 6, 6 times
2 times 3 are 6	2 in 6, 3 times
3 times 2 are 6	3 in 6, 2 times

Lesson VII.

About the Number Seven.

What lesson is this? What is it about? How many lessons have preceded this? How many days in a week? Name them. Make 7 **a**'s on the board. Take 7 books and make two piles of them. What are the two piles called? *Ans.* Parts of the whole. Can you put 6 books in one pile and 1 book in the other? 5 in one and 2 in the other? 3 in one and 4 in the other?

What are the **two parts** of 7? *Ans.* { 1 and 6, 2 and 5, 3 and 4.

Addition.—*Uniting the parts.*

How many are 6 books and 1 book? 1 **b** and 6 **b**'s? 2 **b**'s and 5 **b**'s? 3 **b**'s and 4 **b**'s? 5 **b**'s and 2 **b**'s? 4 **b**'s and 3 **b**'s?

What are the **two parts** of 7? When you unite them, do they make 7? How many are 5 girls and 2 girls? 4 pins and 3 pins? 1 hog and 6 hogs? 3 mugs and 4 mugs?

Ann has 3 prunes and John has 4 prunes; how many have both? 2 hogs are in one pen and 5 hogs are in another; how many in both pens?

Copy and read: $6+1=7$; $4+3=7$; $2+5=7$; $5+2=7$; $3+4=7$; $1+6=7$.

SUBTRACTION.—*Taking away one part.*

Seven books are together, how many would be left if 1 book were taken away? If 4 books were taken away? If 2 books? If 6 books? 3 books? 5 books? 7 books?

What are the **two parts** of 7? When one of them is taken from 7, is the other left? How many are 7 books less 1 book? 7 walnuts less 3 walnuts? 7 pins less 5 pins? 7 nuts less 7 nuts? 7 **n**'s less 2 **n**'s? 7 **n**'s less 4 **n**'s? 7 **n**'s less 6 **n**'s? Moses caught 7 fishes and John caught 4 fishes; how many more fishes did Moses catch than John? Lucy is 7 years old and Betty is 5; how much older is Lucy than Betty?

Copy and read: $7-1=6$; $7-3=4$; $7-5=2$; $7-7=0$; $7-2=5$; $7-4=3$; $7-6=1$.

MULTIPLICATION.—*Uniting equal parts.*

How many are $1+1+1+1+1+1+1$? How many times is 1 taken? How many, then, are 7 times 1? How many are 7 times 1 book? Show me 7 planks. Now show me 7 times 1 plank. Ann has 1 rose and Mary has 5 roses; how many more roses does Mary need to have 7 times as as many as Ann? Are 4 **r**'s and 3 **r**'s more than 7 times 1 **r**? How many do 6 cows lack of being 7 times 1 cow? How many do 3 cows lack?

Copy and read: $7\times 1=7$; $7\times 0=0$.

DIVISION.—*Measuring by a part.*

Does a pile of 7 books contain its parts? Is 1 book one of its parts? How many times is one book con-

tained in the pile? How many times do 7 books contain 1 book? How many times is 1 ox contained in 7 oxen? 1 day in 7 days? 1 foot in 7 feet?

Copy and read: $7 \div 1 = 7$.

Object Exercises.

Take 7 blocks; put them on a table, and call them pies. What stands for pie? *Ans.* **p**.

Now separate the pies into **two parts,** every way you can, calling the results thus: 1 **p** and 6 **p**'s are 7 **p**'s, etc.

Next *take off* 1 pie at a time, calling the results thus: 1 **p** from 7 **p**'s leaves 6 **p**'s; 2 **p**'s from 7 **p**'s leave 5 **p**'s, etc.

Now *put on* 1 pie at a time, calling the results thus: 1 time 1 **p** is 1 **p**; 2 times 1 **p** are 2 **p**'s, etc.

Now *take off* 1 pie at a time, calling the results thus: 1 pie contains 1 pie 1 time; 2 **p**'s contain 1 **p** 2 times, etc.

Learn and recite:

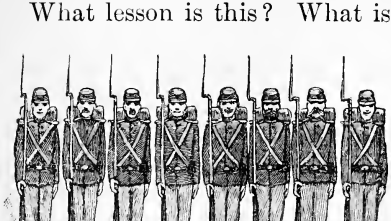
The Table of Seven.

0 and 7 are 7	0 from 7 leaves 7
1 and 6 are 7	1 from 7 leaves 6
2 and 5 are 7	2 from 7 leaves 5
3 and 4 are 7	3 from 7 leaves 4
4 and 3 are 7	4 from 7 leaves 3
5 and 2 are 7	5 from 7 leaves 2
6 and 1 are 7	6 from 7 leaves 1
7 and 0 are 7	7 from 7 leaves 0
7 times 1 are 7	1 in 7, 7 times.

Lesson VIII.

About the Number Eight.

What lesson is this? What is it about? How many lessons have preceded this? In the picture is a squad of soldiers; how many soldiers are in the whole squad? If the squad were divided into **two parts**, how many soldiers would be in each part? *Ans.* 1 soldier in one part and 7 soldiers in the other, or 2 soldiers and 6 soldiers, or 3 soldiers and 5 soldiers, or 4 soldiers and 4 soldiers. How many sets of **two parts** are there? Take 8 books; call them soldiers, and separate them into 4 sets of **two parts.**



What are the **two parts** of 8? *Ans.* { 1 and 7, 3 and 5. 2 and 6, 4 and 4.

Addition.—*Uniting the parts.*

How many are 7 soldiers and 1 soldier? 1 **s** and 7 **s**'s? 3 **s**'s and 5 **s**'s? 6 **s**'s and 2 **s**'s? 4 birds and 4 birds? 5 **b**'s and 3 **b**'s? 2 **b**'s and 6 **b**'s?

What are the **two parts** of 8? What do they make when united or put together? How many are 5 marbles and 3 marbles? 4 **m**'s and 4 **m**'s? 6 cakes and 2 cakes? 7 buds and 1 bud? How many is 1 more than 7? 3 more than 5? 4 more than 4? Ike has 5 peaches, and Jim has three peaches more than Ike; how many peaches has Jim? Ben jumped 6 feet and Tom jumped 2 feet further than Ben; how far did Tom jump?

Copy and read: $1+7=8$; $3+5=8$; $5+3=8$; $7+1=8$; $2+6=8$; $4+4=8$; $6+2=8$; $8+0=8$.

SUBTRACTION.—*Taking away one part.*

How many soldiers would be left in the squad if 1 soldier were taken away? If two soldiers were taken away? If 3 soldiers? If 4 soldiers? 5 soldiers? 6 soldiers? 7 soldiers? 8 soldiers?

What are the **two parts** of 8? When one of them is taken from 8, what is left? I wish to put 8 birds in two cages so as to have 3 birds in one cage, how many birds will be in the other cage? How can I put 8 hogs in two pens so as to have 2 hogs in one of the pens? Moses and Joseph ate 8 biscuits together; if Joseph ate 4 biscuits, how many did Moses eat? Which is the more, 8 or 5? How many more?

Copy and read: $8-2=6$; $8-3=5$; $8-6=2$.

MULTIPLICATION.—*Uniting equal parts.*

Make 8 marks. How many times did you make 1 mark? 8 times 1 mark are how many? Are 2 marks a part of 8 marks? If you make 2 marks 4 times, thus: //, //, //, //, how many marks will there be in all? How many, then, are 4 times 2 marks? 4 times 2 soldiers? Are 4 marks a part of 8 marks? If you make 4 marks 2 times, thus: ////, ////, how many marks will there be in all? How many, then, are 2 times 4 marks? 2 times 4 soldiers?

Ike has 4 marbles and John has 2 times as many as Ike; how many marbles has John? Harry has 4 times 2 dimes and Jane has 2 times 4 dimes which has the more?

Copy and read: $8\times1=8$; $4\times2=8$; $2\times4=8$.

DIVISION. — *Measuring by a part.*

Does 8 contain its parts? Does the squad of 8 soldiers contain 1 soldier 8 times? Suppose 2 soldiers were taken away; then 2 soldiers more were taken away; then 2 more; then 2 more; would there be any soldiers left? How many times can 2 soldiers be taken away? How many times, then, do 8 soldiers contain 2 soldiers? If 4 soldiers were taken away, and then 4 soldiers more, would any soldiers remain? How many times can 4 soldiers be taken away? How many times, then, do 8 soldiers contain 4 soldiers?

OBJECT EXERCISES.

Take 8 blocks; put them on a table and call them soldiers. What stands for soldier? *Ans.* **s.**

Now separate the soldiers into **two parts** every way you can, calling the results thus: 1 **s** and 7 **s**'s are 8 **s**'s, etc.

Now *take off* 1 soldier at a time, calling the results thus: 1 **s** from 8 **s**'s leaves 7 **s**'s; 2 **s**'s from 8 **s**'s leave 6 **s**'s, etc.

Now *put on* 1 soldier at a time, calling the results thus: 1 time 1 **s** is 1 **s**; 2 times 1 **s** are 2 **s**'s, etc.

Now *take off* 1 **s** at a time, calling the results thus: 1 **s** contains 1 **s**, 1 time; 2 **s**'s contain 1 **s**, 2 times, etc.

Next *put on* 2 soldiers at a time, calling the results thus: once 2 **s**'s is 2 **s**'s; 2 times 2 **s**'s are 4 **s**'s, etc.

Next *take off* 2 soldiers at a time, calling the results thus: 2 **s**'s contain 2 **s**'s, 1 time; 4 **s**'s contain 2 **s**'s, 2 2 times, etc.

Now *put on* 4 soldiers at a time, etc., and then *take off* 4 **s**'s at a time, etc.

Learn and recite:

The Table of Eight.

0 and 8 are 8	0 from 8 leaves 8
1 and 7 are 8	1 from 8 leaves 7
2 and 6 are 8	2 from 8 leaves 6
3 and 5 are 8	3 from 8 leaves 5
4 and 4 are 8	4 from 8 leaves 4
5 and 3 are 8	5 from 8 leaves 3
6 and 2 are 8	6 from 8 leaves 2
7 and 1 are 8	7 from 8 leaves 1
8 and 0 are 8	8 from 8 leaves 0
1 time 8 is 8	1 in 8, 8 times
2 times 4 are 8	2 in 8, 4 times
4 times 2 are 8	4 in 8, 2 times

Lesson IX.

About the Number Nine.

Is 9 one more than 8? Hold up 9 fingers. Make 9 steps and count them as you step.

a a a a a a a a a

How many **a**'s are in the above row? Count and see if there are 8 **a**'s and 1 **a**? 7 **a**'s and 2 **a**'s? 6 **a**'s and 3 **a**'s? 5 **a**'s and 4 **a**'s?

What are the **two parts** of 9? *Ans.* { 8 and 1, 7 and 2, 6 and 3, 5 and 4.

ADDITION.—*Uniting the parts.*

Count 9. How many are 8 men and 1 man? 7 **m**'s and 2 **m**'s? 6 cats and 3 cats? 5 **c**'s and 4 **c**'s? What are the **two parts** of 9? When united do they make 9? Is 1 more than 8 the same as 2 more than 7? Is 3 more than 6 the same as 4 more than 5? How many are 4 and 5? 5 and 4? 6 and 3? 3 and 6? 7 and 2? 2 and 7?

Five eggs in one nest and 4 eggs in another; how many eggs in all? 7 tulips and 2 tulips are how many? 6 daisies in one cluster and 3 in another; how many daisies in both clusters?

Copy and read: $1+8=9$; $2+7=9$; $3+6=9$; $5+4=9$.

SUBTRACTION.—*Taking away one part.*

Count 9 backward. How many are 2 ducks less than 9 ducks? 4 doves less than 9 doves? 6 rats less than 9 rats? What are the **two parts** of 9? When one of them is taken from 9, what is left? 9 birds were on a limb, but three of them have flown, how many birds are left? The old hen has 9 chicks; 6 of the chicks are on one side of the fence, how many are on the other side? A boy had 9 marbles but lost 5 of them, how many marbles had he left? If 7 chairs are 1 part of a set of 9 chairs, how many are in the other part? 7 from 9 leaves how many?

Copy and read: $9-1=8$; $9-2=7$; $9-3=6$; $9-4=5$; $9-5=4$.

MULTIPLICATION.—*Uniting equal parts.*

Make 9 **a**'s. How many times did you make 1 **a**? How many, then, are 9 times 1 **a**? 9 times 1 horse?

9 times 1 boy? Are 3 **a**'s a part of 9 **a**'s? If you make 3 **a**'s 3 times, thus: **aaa**, **aaa**, **aaa**, how many **a**'s will there be in all? How many, then, are three times 3? 3 times 3 cows? 3 times 3 cats? Show me 3 times 3 fingers. Show me 3 times 3 planks. Little Hal is 3 years old, and Mattie is three times as old as Hal; how old is Mattie? Are 9 times 1 dollar more than 3 times 3 dollars?

Copy and read: $9 \times 1 = 9$; $3 \times 3 = 9$.

DIVISION.—*Measuring by a part.*

Does 9 **a** contain its parts? Is 1 **a** one of the parts? How many **a**'s does a row of 9 **a**'s contain? Make 9 **a**'s. Erase 3 **a**'s at a time, until all are gone. How many times did you erase? How many times, then, are 3 **a**'s contained in 9 **a**'s?

Is a yard-stick 3 feet long? How many times is a yard-stick contained in 9 feet? What is a yard-stick for? *Ans.* For measuring. Can I measure a pole, 9 feet long, by it? How many measures would it take? How many does 7 lack of containing one 9 times? How many does 5 lack? 6? 8? 3? How many do 8 cups lack of containing 3 cups 3 times? How many do 4 cups lack?

Copy and read: $9 \div 1 = 9$; $9 \div 3 = 3$; $9 \div 9 = 1$.

OBJECT EXERCISES.

Take 9 blocks, put them on the table and call them horses. What stands for horses? *Ans.* **h**.

Now separate the horses into **two parts** every way you can, calling the results thus: 1 **h** and 8 **h**'s are 9 **h**'s, etc.

Next *take off* 1 horse at a time, calling the results thus: 1 horse from 9 horses leaves 8 horses; 2 **h**'s from 9 **h**'s leave 7 **h**'s, etc.

Next *put on* one horse at a time, calling the results thus: 1 time 1 **h** is 1 **h**; 2 times 1 **h** are 2 **h**'s, etc.

Next *take off* 1 horse at a time, calling the results thus: 1 **h** contains 1 **h**, 1 time; 2 **h**'s contain 1 **h**, 2 times, etc.

Next *put on* 3 horses at a time, calling the results thus: 1 time 3 **h**'s is 3 **h**'s; 2 times 3 **h**'s are 6 **h**'s, etc.

Now *take off* 3 **h**'s at a time, calling the results thus: 3 **h**'s contain 3 **h**'s, 1 time; 6 **h**'s contain 3 **h**'s, 2 times, etc.

NOTE.—The teacher may repeat the object exercises, giving the blocks or pebbles such names as will interest and amuse the pupils.

Learn and recite—

THE TABLE OF NINE.

0 and 9 are 9	0 from 9 leaves 9
1 and 8 are 9	1 from 9 leaves 8
2 and 7 are 9	2 from 9 leaves 7
3 and 6 are 9	3 from 9 leaves 6
4 and 5 are 9	4 from 9 leaves 5
5 and 4 are 9	5 from 9 leaves 4
6 and 3 are 9	6 from 9 leaves 3
7 and 2 are 9	7 from 9 leaves 2
8 and 1 are 9	8 from 9 leaves 1
9 and 0 are 9	9 from 9 leaves 0
1 time 9 is 9	1 in 9, 9 times
3 times 3 are 9	3 in 9, 3 times

Lesson X.

About the Number Ten.

What lesson is this? What is it about? 10 is the next number after what? Count all your fingers. How many have you? How many frogs in the picture? How many are on the log? How many are on the ground? If another frog gets on the log, how many would then be on the log and ground? How many if 2 frogs more get on the log? How many if 3 frogs more get on the log? 4 frogs more? 5 frogs? Is *one part* of 10 frogs on the log and the *other part* on the ground?

What are the **two parts** of 10? *Ans.* { 1 and 9, 3 and 7, 2 and 8, 4 and 6, 5 and 5.

Addition. — *Uniting the parts.*

How many are 7 frogs and 3 frogs? 8 **f**'s and 2 **f**'s? 9 **f**'s and 1 **f**? 4 men and 6 men? 5 **m**'s and 5 **m**'s? 3 birds and 7 birds? 2 **b**'s and 8 **b**'s? What are the **two parts** of 10? How many do they make when put together? How many are 5 marbles and 5 marbles? 7 eggs and 3 eggs? 6 cakes and 4 cakes?

Charles is 7 years old and Fannie is 3 years older; how old is Fannie? Mary has 6 roses and Jane has 4 roses; how many roses have both girls?

Copy and read: $4 + 6 = 10$; $8 + 2 = 10$; $3 + 7 = 10$.

SUBTRACTION.— *Taking away one part.*

If 10 frogs were on a log, and 1 frog should leap off, how many frogs would be left? How many would be left if 2 frogs should leap off? If 3 frogs should leap off? If 4 frogs? 5 frogs? 6 frogs? 7 frogs? 8 frogs? 9 frogs? 10 frogs?

What are the **two parts** of 10? When one of them is taken from 10, what is left? How many are 8 and 2? How many, then, is 10 less 2? There are 10 frogs in all; if 4 frogs are on the log, how many are on the ground?

Ann had 10 little birds, but the old cat ate 3 of them; how many birds has Ann now? 10 pinks in one bed, and 6 pinks in another bed; how many more pinks in one than in the other? 10 hogs were in the garden, but John turned 4 hogs out; how many remain in the garden?

Copy and read: $10-3=7$; $10-5=5$; $10-8=2$.

MULTIPLICATION.— *Uniting equal parts.*

Count 10. Make 10 **c**'s. How many times did you make 1 **c**? How many, then, are 10 times 1 **c**? 10 times 1 horse? Are 2 **c**'s a part of 10 **c**'s? If you make 2 **c**'s 5 times, thus: **cc**, **cc**, **cc**, **cc**, **cc**; how many **c**'s will there be in all? How many, then, are 5 times 2 **c**'s? 5 times 2 cups? 5 times 2 caps? If you make 5 **c**'s 2 times, how many **c**'s will there be in all? How many, then, are 2 times 5 **c**'s? 2 times 5 cats? Are 5 times 2 dollars the same as 2 times 5 dollars? Little Ella is 5 years old, and her brother is 2 times as old as she; how old is her brother?

Copy and read: $10\times 1=10$; $5\times 2=10$; $2\times 5=10$.

DIVISION.—*Measuring by a part.*

Does 10 contain its parts? How many 1's in 10? How many times, then, does 10 contain 1? Make 10 **c**'s. Now erase 2 **c**'s at a time until all are gone; how many times did you erase? How many times, then, do 10 **c**'s contain 2 **c**'s? 10 caps contain 2 caps? 10 cabs contain 2 cabs? Show me 10 fingers. Now remove 5 fingers at a time; how many times did you remove? How many times, then, do 10 fingers contain 5 fingers? 10 fobs contain 5 fobs? 10 fans contain 5 fans?

Copy and read: $10 \div 1 = 10$; $10 \div 2 = 5$; $10 \div 5 = 2$.

OBJECT EXERCISES.

Put 10 blocks on a table and call them lions. What stands for lions? *Ans.* **l.**

Now separate the lions into **two parts** every way you can, calling the results thus: 1 **l** and 9 **l**'s are 10 **l**'s, etc.

Now *take off* 1 lion at a time, calling the results thus: 1 **l** from 10 **l**'s leaves 9 **l**'s; 2 **l**'s from 10 **l**'s leave 8 **l**'s, etc.

Now *put on* 1 lion at a time, calling the results thus: 1 time 1 **l** is 1 **l**; 2 times 1 **l** are 2 **l**'s, etc.

Next *take off* 1 lion at a time, calling the results thus: 1 **l** contains 1 **l**, 1 time; 2 **l**'s contain 1 **l**, 2 times, etc.

Next *put on* 2 lions at a time, calling the results thus: 1 time 2 **l**'s is 2 **l**'s; 2 times 2 **l**'s are 4 **l**'s, etc.

Now *take off* 2 lions at a time, calling the results thus: 2 **l**'s contain 2 **l**'s 1 time; 4 **l**'s contain 2 **l**'s 2 times, etc.

The Table of Ten.

0	and	10	are	10	
1	and	9	are	10	
2	and	8	are	10	
3	and	7	are	10	
4	and	6	are	10	
5	and	5	are	10	
6	and	4	are	10	
7	and	3	are	10	
8	and	2	are	10	
9	and	1	are	10	
1	time	10	is	10	
2	times	5	are	10	
5	times	2	are	10	

0	from	10	leaves	10	
1	from	10	leaves	9	
2	from	10	leaves	8	
3	from	10	leaves	7	
4	from	10	leaves	6	
5	from	10	leaves	5	
6	from	10	leaves	4	
7	from	10	leaves	3	
8	from	10	leaves	2	
9	from	10	leaves	1	
1	in	10,	10	times	
2	in	10,	5	times	
5	in	10,	2	times	

Review Questions.

Mental Exercises.

What is Addition? *Ans.* Uniting parts or numbers.

What is the sign of Addition? *Ans.* +. What is it called? *Ans.* And.

What is the number whose ***two parts*** are 4 and 3? 1 and 6? 2 and 5? 7 and 3? 2 and 2? 1 and 8? 3 and 2? 2 and 7? 5 and 4? 2 and 6? 6 and 3? 5 and 4? 3 and 3? 4 and 2? 3 and 5? 8 and 2? 4 and 4? 5 and 5? 6 and 4?

How many are 4 birds and 3 birds? 1 **b** and 2 **b**'s? 2 **b**'s and 5 **b**'s? 7 cats and 3 cats? 2 **c**'s + 7 **c**'s? 4 hats and 4 hats? 2 **h**'s and 6 **h**'s? 4 men and 2 men? 3 **m**'s and 5 **m**'s?

What is Subtraction? *Ans.* Taking away one part of a number, or taking one number from another.

What is the sign of Subtraction? *Ans.* —. What is it called? *Ans.* Less.

Two is one part of the number 3, what is the other part? What is the other part of 7 if 3 is one part? If 6? What

is the other part of 9 if 5 is one part? If 7? If 3? What is the other part of 8 if 6 is one part? If 3? If 4?

How many are 7 birds less 3 birds? 7 **b**'s less 5 **b**'s? 9 caps less 1 cap? 8 **c**'s — 6 **c**'s? 9 tubs less 3 tubs? 6 **t**'s less 3 **t**'s? 10 fans less 3 fans? 10 **f**'s — 5 **f**'s?

What is the difference between 7 and 2? 10 and 3? 9 and 4? 8 marbles and 5 marbles? 5 **m**'s and 3 **m**'s? 10 rats and 8 rats? 9 rats and 5 rats? 7 **r**'s and 4 **r**'s?

What is Multiplication? *Ans.* Uniting equal parts or numbers. What is the sign of Multiplication? *Ans.* $\times$. What is it called? *Ans.* Times.

How many are 1 taken 5 times? 1 taken 6 times? 3 taken 2 times? 2 taken 3 times? 5 taken 2 times? 2 taken 5 times? 2 taken 2 times? 4 taken 2 times? 2 taken 4 times? 0 taken 7 times? 7 taken 0 times?

How many are 9 times 1 bird? 7 times 1 **b**? 1 time 5 birds? 6 times 1 hat? 3×2 **h**'s? 10 times 1 box? 5×2 **b**'s? 3×3 **b**'s? 4×2 **b**'s? 7 times 1 watch? 2×4 **w**'s? 2×2 **w**'s?

What is Division? *Ans.* Measuring a number by one of its parts, or by another number. What is the sign of Division? *Ans.* $\div$. What is it called? *Ans.* Contains.

How many times does 5 contain 1? 7 contain 1? 10 contain 2? 8 contain 4? 6 contain 2? 4 contain 2? 6 contain 6? 6 contain 3? 8 contain 2? $10 \div 5$? $9 \div 3$?

Slate Exercises.

Copy and add—

1	2	2	3	1	2	3	4	5	6	3	3
1	1	2	2	4	5	6	2	3	2	7	4
1	1	2	3	2	3	5	6	7	8	6	3
2	3	1	2	2	0	1	3	1	1	4	3
3	5	6	5	4	7	3	1	2	0	0	3

2	1	1	3	2	4	1	3	2	1	1	1
2	3	0	1	3	1	0	3	2	4	1	0
0	1	4	1	1	3	6	3	2	4	2	3
6	2	4	5	1	0	3	1	2	1	6	5

Copy and subtract—

2	3	4	5	5	7	9	6	8	5	4	7
1	2	2	3	1	3	5	4	3	2	1	5

7	10	9	6	8	5	7	9	10	8	6	9
2	6	3	3	2	4	4	4	7	6	2	6

9	6	8	10	7	9	10	8	6	10	9	8
7	6	5	5	6	8	3	7	5	2	2	4

Copy and multiply—

5	3	2	4	7	8	1	4	2	1	3	5
1	1	2	1	1	1	7	2	3	8	3	2

2	9	2	3	1	7	8	1	0	5	2	4
5	1	4	2	6	0	1	4	9	2	3	2

Copy and divide—

1)5	2)6	7)7	2)4	3)6	2)10	1)9	6)6	1)10	2)8

1)8	4)8	5)5	3)3	2)6	2)8	2)10	3)9	4)8	5)10

PART II.

NOTATION AND NUMERATION.

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1. **A Unit** is a single thing or one; as *one*, *one* dime, *one* dozen, *one* hundred.

2. **A Number** is a unit or a collection of units; as *one*, *five*, *seven* men, *twenty* days.

3. **The Unit of a Number** is one of the units of which the number is formed. Thus, the unit of *nine* is *one;* of *twelve* quarts, *one* quart; of *eleven* dozen, *one* dozen.

4. An **Abstract Number** is a number that is not applied to any particular object; as *three*, *ten*, *fifteen.*

5. **A Concrete Number** is a number that is applied to some particular object; as *two* cents, *six* bushels, *twenty-five* dollars.

6. EXERCISES. — What is the unit of *Four? Five* boys? *Nine? Twelve* dollars? *Twenty* days? Which of these numbers are abstract and which are concrete? Name five abstract, also five concrete numbers.

7. **Notation** is the art of expressing numbers by symbols.

8. **Numeration** is the art of reading numbers that are expressed by symbols.

9. **Symbols** may be either letters or figures.

Numbers from 0 to 10.

10. In the **Arabic Notation** ten figures are used to express numbers, viz:

Figures—*0, 1, 2, 3, 4, 5, 6, 7, 8, 9.*
Names—**Naught, One, Two, Three, Four, Five, Six, Seven, Eight, Nine.**

11. The figure 0, also called cipher or zero, denotes *none* or *nothing.* The other nine figures are called digits.

12. The largest number that can be expressed by a single figure is *nine* or 9. Nine and one are ten, which is written 10.

Numbers from 10 to 100.

a a a a a a a a a a	a	10th Row.
a a a a a a a a a a	a	9th "
a a a a a a a a a a	a	8th "
a a a a a a a a a a	a	7th "
a a a a a a a a a a	a	6th "
a a a a a a a a a a	a	5th "
a a a a a a a a a a	a	4th "
a a a a a a a a a a	a	3d "
a a a a a a a a a a	a	2d "
a a a a a a a a a a	a	1st "
Column of Tens.	Column of Units.	

13. How many **a**'s in each row of units? *Ans. one.*
How many **a**'s in each row of tens? *Ans.* 1 *ten.*
How many **a**'s in 2 rows of tens? *Ans.* 2 tens.
How many **a**'s in 3 rows of tens? In 4 rows of tens? In 5 rows of tens? In 6? In 7? 8? 9? 10?

Count the **a**'s in the first 2 rows of tens; in the first 3 rows of tens; in the first 4 rows; in the first 5 rows, etc., etc.

How many tens in twenty? In thirty? Forty? Fifty? Sixty? Seventy? Eighty? Ninety? One hundred?

How many units in 2 tens? In 3 tens? In 4 tens? Etc., etc.

Count the **a**'s in 2 rows of tens and 7 rows of units. What is the short method of denoting the number of **a**'s in "2 rows of tens and 7 rows of units?" *Ans.* 27 **a**'s. Point out the rows that you would count to get 42 **a**'s; 57 **a**'s; 63 **a**'s; 91 **a**'s; 35 **a**'s; 17 **a**'s; 20 **a**'s.

14. When a number is expressed by two figures, the figure at the right denotes *units* or *ones*, and the figure at the left denotes *tens*.

By counting, show that:

10	stands	for	Ten.	16	stands	for	Sixteen.
11	"	"	Eleven.	17	"	"	Seventeen.
12	"	"	Twelve.	18	"	"	Eighteen.
13	"	"	Thirteen.	19	"	"	Nineteen.
14	"	"	Fourteen.	20	"	"	Twenty.
15	"	"	Fifteen.	21	"	"	Twenty-one.

What do the figures 57 denote?

Ans. 5 tens 7 ones; read *fifty-seven.*

What do the figures 83 denote?

Ans. 8 tens 3 ones; read *eighty-three.*

15. Copy and read the following numbers, naming the *tens* and *units* in each:

23.	47.	13.	63.	73.	36.	19.
42.	58.	24.	35.	78.	27.	33.
71.	85.	78.	40.	99.	31.	55.
93.	60.	81.	49.	88.	69.	100.

16. Express by figures:

1. Seventy-three.	9. Seventy-seven	17. Twenty-seven.
2. Sixty-nine.	10. Ninety.	18. Forty-five.
3. Eighty-eight.	11. Seventy-eight.	19. Seventy-one.
4. Forty-nine.	12. Thirty-nine.	20. Sixty-seven.
5. Ninety-nine.	13. Ninety-six.	21. Eighty-one.
6. Fifty-six.	14. Eighty-four.	22. Sixty-six.
7. Eighty-three.	15. Fifty-nine.	23. Twenty-nine.
8. Forty-four.	16. Forty-seven.	24. One-hundred.

17. The greatest number that can be expressed by two figures is *ninety-nine*, or 99, which represents 9 *tens* and 9 *units*. 99 and 1 more make *one hundred*, written 100. 100 is 10 *tens*.

Numbers from 100 to 1000.

18. *Ten* rows of *ten* **a**'s, arranged as on page 42, represent the numeral frame. How many **a**'s are in *one* frame? *Ans.* One hundred.

How many in 2 frames? *Ans.* Two hundred.

How many in 3 frames? In 4 frames? In 5 frames? etc. How many **a**'s are in 3 frames 5 rows of *tens* and 7 rows of *ones?* *Ans.* 3 *hundreds* 5 *tens* 7 *ones*, which is written 357, and read *three hundred fifty-seven.*

19. When a number is expressed by three figures, the figure at the right denotes *units;* the middle figure, *tens;* and the figure at the left, *hundreds.*

One hundred,	or 1	hd.	0	tens,	0	ones,	is denoted by	100
Two hundred,	" 2	"	0	"	0	"	"	200
Three hundred,	" 3	"	0	"	0	"	"	300
Four hundred,	" 4	"	0	"	0	"	"	400
Five hundred,	" 5	"	0	"	0	"	"	500
Six hundred,	" 6	"	0	"	0	"	"	600
Seven hundred,	" 7	"	0	"	0	"	"	700
Eight hundred,	" 8	"	0	"	0	"	"	800
Nine hundred,	" 9	"	0	"	0	"	"	900

What do the figures 345 denote? *Ans.* 3 *hund.* 4 *tens* 5 *ones*; read, three hundred forty-five.

20. Read the following numbers, naming the hundreds, *tens*, and *units* of each:

107.	538.	297.	580.	777.	896.
756.	627.	268.	579.	180.	403.
865.	921.	354.	412.	141.	501.
494.	330.	945.	813.	754.	999.

21. Express by *figures:*

1. Two hundred twenty-two.
2. Three hundred thirty-three.
3. Eight hundred five.
4. Five hundred twenty-seven.
5. Seven hundred forty-one.
6. Nine hundred seventy-four.
7. Four hundred ninety-nine.
8. Six hundred sixty-six.
9. Four hundred fifty.
10. Nine hundred eighty-one.
11. Seven hundred sixty-seven.
12. Six hundred thirty-two.
13. One hundred two.

14. Three hundred fifteen.
15. Two hundred twelve.
16. Nine hundred ninety-nine.

22. The greatest number that can be expressed by three figures is nine hundred ninety-nine, or 999, which represents 9 *hunds.* 9 *tens* 9 *units.* 999 and 1 more make one thousand; written, 1000. 1000 is 10 *hundreds.*

Numbers from 1000 to 10000.

23. If we call *ten* "numeral frames" a *pack*, how many **a**'s are in one pack? *Ans.* One thousand.

How many **a**'s in 2 packs? *Ans.* Two thousand.

How many in 3 packs? In 4 packs? In 5 packs? etc.

How many **a**'s in 7 packs, 3 frames, 2 rows of tens, and 5 rows of ones?

Ans. 7 thousands 3 hundreds 2 tens 5 ones, which is written 7325, and read, seven thousand three hundred twenty-five.

What do the figures 4609 denote?

Ans. 4 thousands 6 hundreds 0 tens 9 ones; read, four thousand six hundred nine.

What do the figures 7040 denote?

Ans. 7 thousands 0 hundreds 4 tens 0 ones; read, seven thousand forty.

What do the figures 9003 denote?

Ans. 9 thousands 0 hundreds 0 tens 3 ones; read, nine thousand three.

24. Copy and read:

1974.	3541.	4200.	7532.	8007.
8013.	5727.	2975.	5380.	6483.
6304.	7090.	9756.	3868.	9999.

25. Express by figures:

1. One thousand four hundred seventy-five.
2. Three thousand two hundred nineteen.
3. Five thousand seven hundred twenty-seven.
4. Seven thousand three hundred fifty-four.
5. Two thousand six hundred twelve.
6. Eight thousand one hundred forty.
7. Nine thousand nine hundred ninety-nine.
8. Nine thousand six.
9. Four thousand ten.
10. Six thousand eleven.
11. One thousand fifteen.
12. Five thousand fifty.
13. Seven thousand one.

Writing and Reading Numbers in General.

Places and Periods.

26. In any number the **places** or **orders** of the figures are numbered from the right. Thus, in 3425867,

7 is in the 1st **place,**
6 " " " 2d "
8 " " " 3d "
5 " " " 4th "
2 is in the 5th **place,**
4 " " " 6th "
3 " " " 7th "
etc., etc.

27. The **periods,** which consist of three figures each, are also numbered from the right. Thus, in 345,896,701,

701 is the 1st **period.**
896 " " 2d "
345 " " 3d "

Exercises.

28. In 42375809, what figure is in the 2d place? In the 5th? 7th? 1st? 3d? Of what *order* is 8?

Ans. 3d order.

Mention the order of each figure. Separate the number into *periods* by commas. Which is the 1st period? The 2d? 3d? How many figures in the 3d period? What is the unit of 7? 9 *cents?* 5 *tens?* What is the value of 5 if the unit is one peck? *Ans.* 5 pecks.

What is the value of 5 if the unit is 1 *ten?*

Ans. 5 *tens* or 50.

What is the value of 7 if the unit is *one?* 1 *ten?* 1 *hundred?* 1 *thousand?*

What is the unit of 4 in 43? *Ans.* 1 *ten?*

What is the *local* value of 4? *Ans.* 4 tens or 40.

In 3725 give the unit and local value of each figure.

29. Table of Tens.

10 Units	make	1 Ten	10
10 Tens	"	1 Hundred	100
10 Hundreds	"	1 Thousand	1000
10 Thousands	"	1 Ten-thousand . . .	10000
10 Ten-thousands	"	1 Hundred-thousand .	100000

30. Table of Thousands.

1000 Units	make	1 Thousand . .	1,000
1000 Thousands	"	1 Million . . .	1,000,000
1000 Millions	"	1 Billion . . .	1,000,000,000

31. Table of Places or Orders.

A figure in the							
1st	place denotes	**Ones**	called units of the		1st	**order.**	
2d	"	"	**Tens**	"	"	2d	"
3d	"	"	**Hundreds**	"	"	3d	"
4th	"	"	**Thousands**	"	"	4th	"
5th	"	"	**Ten-thousands**	"	"	5th	"
6th	"	"	**Hundred-thousands**	"	"	6th	"

32. Table of Periods.

The 1st period represents *Ones.*
2d " " Thousands.
3d " " Millions.
4th " " Billions.
5th " " Trillions.

33. Numeration Table.

	15th. Hundreds.	14th. Tens.	13th. Ones.	12th. Hundreds.	11th. Tens.	10th. Ones.	9th. Hundreds.	8th. Tens.	7th. Ones.	6th. Hundreds.	5th. Tens.	4th. Ones.	3d. Hundreds.	2d. Tens.	1st. Ones.
Places of units															
Figures	3	6	0,	5	0	4,	7	2	5,	9	1	3,	8	7	6.
Periods — Numbers	Fifth.			Fourth.			Third.			Second.			First.		
Periods — Names	Trillions.			Billions.			Millions.			Thousands.			Ones.		

Exercises in Numeration.

34. 1. Read the number represented by 37000401064.

Operation.—Separate the number into periods, thus: 37,000, 401,064. The 4th period is billions, the 3d is millions, the 2d is thousands, the 1st is units; hence, the number is 37 billion 0 million 401 thousand 64 units, or thirty-seven billion four hundred one thousand sixty-four.

Rule.—I. *Begin at the right and separate the number into periods.*

II. *Then begin at the left and read each successive period as if it stood alone, giving each its name except the period of units.*

In this manner read:

2.	43564.	8.	54311237.
3.	75031.	9.	801603709.
4.	132140.	10.	4321780651.
5.	5720307.	11.	123456789.
6.	4006009.	12.	123456654321.
7.	7205806.	13.	230405060708090.

Exercises in Notation.

35. 1. Express in figures fifty-three billion sixty-five million three hundred seven.

Operation.—Since billions, the highest number named, occupy the fourth period, there will be four periods in the number. Now, beginning at the left, we fill each of the periods with the given numbers of *billions*, *millions*, *thousands*, and *units*, as if each stood alone, and obtain

53,065,000,307.

Rule I.—*Consider from the greatest number named the necessary number of periods.*

II.—*Begin at the left and fill each of the successive periods as if it stood alone.*

Note.—There must be three figures in every period, except the one at the left, which may have one, two, or three. All vacant orders and periods must be filled with ciphers.

In this manner express in figures:

2. Eighteen thousand five hundred thirty-six.
3. Thirty-two thousand eight.
4. Forty-seven thousand two hundred.
5. Two hundred forty thousand five hundred one.

6. Six million five thousand forty-seven.

7. Nine million twenty-three thousand thirty-one.

8. Twenty-nine million four hundred twelve thousand five.

9. One hundred seven million eleven thousand one hundred four.

10. Seven hundred thirty million six hundred nine thousand three hundred ninety-two.

11. Thirteen billion thirteen million thirteen thousand thirteen.

12. Two hundred forty-five billion one hundred seven million fifty nine thousand eight hundred seventy.

Notation of Dollars and Cents.

36. The **Sign of Dollars** is $, which is read, *dollars.*

Thus, $12 is read 12 dollars.

37. The **Sign of Cents** is c, or cts., which is read, *cents.*

Thus, 23c., or 23 cts., is read 23 cents.

38. Dollars and cents may be written as one number by placing a point (.) between them.

Thus, 25 dollars 34 cents, is written $25.34.

39. Since it takes 100 cents to make a dollar, cents always occupy two places at the right of the point. Hence, when the number of cents is less than 10, a cipher must be written between it and the point.

Thus, 5c. is written $.05; and 3 dollars 8 cents is written $3.08. Neither the sign ($) nor the point (.) should be omitted.

40. Exercises in Numeration of Dollars and Cents. Read:

1. $5.35.	5. $17.13.	9. $3140.43.
2. $7.40.	6. $33.07.	10. $504.67.
3. $10.09.	7. $ .34.	11. $5008.03.
4. $.06.	8. $1.01.	12. $.04.

41. Exercises in Notation of Dollars and Cents. Express in figures and signs:

1. Thirteen dollars fifteen cents.
2. Forty dollars fifty cents.
3. Forty-three dollars seven cents.
4. One hundred dollars twenty cents.
5. Sixty dollars ten cents.
6. Thirty-five cents.
7. Nine cents.
8. Eighty-four dollars six cents.
9. Ninety-nine dollars twelve cents.
10. Fifty-four cents.

42. A Scale in Arithmetic is the relation between the successive orders of units.

In the Arabic system of notation, the scale is ten; that is, the value of the unit in any order is ten times as great as the unit in the next lower order; hence, it is called the Decimal Scale, from the Latin word *decem*, meaning ten.

Notation of Objects.

43. In this work objects are frequently denoted by the first letters of their names. Thus, an arm, an apple, etc., is denoted by **a**; a box, a bin, a boy, etc., is denoted by **b**.

Conversely, an **a** may be regarded as denoting an apple, or an ax, etc.; **h** as denoting an hour, a hand, etc. If a letter appears more than once in the same example, each one denotes the same thing. Thus: 2 **b** and 1 **b** are 3 **b**, may be read, 2 boys and 1 boy are 3 boys; or 2 boxes and 1 box are 3 boxes.

Abbreviations.

44. The colon (:) is employed to denote "*the following,*" or "*as follows,*" and to signify that the term or phrase preceding it is to be prefixed to each of the phrases following it. Thus, "from 5 take: 3, 4;" means "from 5 take 3; from 5 take 4."

Questions for Review.

45. What is: 1. A unit? 2. A number? 3. The unit of a number? 4. An abstract number? 5. A concrete number? 6. Notation? 7. Numeration? 8. A scale?

Name: 1. The figures. 2. The digits.

What is the greatest number that can be expressed by: 1. One figure? 2. Two figures? 3. Three figures? 4. Four figures?

How many figures in: 1. One place? 2. One period?

How are places and periods numbered?

Repeat the table of: 1. Tens. 2. Thousands. 3. Places. 4. Periods.

What is the rule for: 1. Reading numbers? 2. Writing numbers?

What stands for: 1. Dollars? 2. Cents? How are dollars and cents written as one number?

In this book: What often stands for an object? What does a colon (:) denote?

ADDITION.

INDUCTIVE EXERCISES.

46. 1. Count 7. *Ans.* 1, 2, 3, 4, 5, 6, 7.

2. Count \$7. *Ans.* \$1, \$2, \$3, \$4, \$5, \$6, \$7.

3. In counting, what are the next 3 numbers above \$4? *Ans.* \$5, \$6, \$7.

4. If I count \$4 and then count \$3 more, how much will I have counted in all? *Ans.* \$7.

5. In counting, what are the next 4 numbers above 6 hats? *Ans.* 7 hats, 8 hats, 9 hats, 10 hats.

6. If I count 6 hats and then count 4 more, how many hats will I have counted in all? *Ans.* 10 hats.

In counting, what is the:	How many are:
7. 1st number above 8?	7. 8 and 1?
8. 2d number above 9?	8. 9 and 2?
9. 3d number above \$5?	9. \$5 and \$3?
10. 4th number above 8 hours?	10. 8 hours and 4 hours?
11. 5th number above 4 pecks?	11. 4 pecks and 5 pecks?

12. James makes 7 steps and then 4 steps more; how many steps does he make in all?

13. If you have 5 apples and I give you 3 more, how many apples will you then have?

14. Seven peaches are on one tree and 6 on another tree; how many peaches on both trees?

15. John paid 8 cents for apples and 4 cents for pears; how much did he spend?

How many are:	How many are:
16. $6 and $1.	16. $1 added to $6.
17. 7 men and 2 men?	17. 2 men added to 7 men?
18. 5 pints and 3 pints.	18. 3 pints added to 5 pints?

Definitions.

47. Like Numbers are those which have units of the same kind, as 7 and 13; 3 pounds and 11 pounds; 6 tens and 20 tens.

48. Unlike Numbers are those which have units of different kinds, as 5 yards and 17 pints; 3 days and 25 feet.

49. Addition is the process of uniting two or more like numbers into *one.*

50. The numbers added are called the **parts,** and the number obtained by adding, the **sum** or **amount.**

Thus, 7 balls and 3 balls, when united, make a group of 10 balls. Here 7 balls and 3 balls are the *parts,* and 10 balls the *sum* or *amount.*

51. A Sign is a symbol used to indicate an operation or relation.

52. The Sign of Addition is +, which is read: *and* or *plus;* plus means more. When + stands between two numbers, it indicates that they are to be added.

53. The Sign of Equality is =, which is read: *are* or *equals.*

54. The Sign of Interrogation is ?, which is read: *what* or *how many.* It signifies that the answer is to be found, and when found, belongs in the place occupied by the sign.

Thus, $9 + 5 = 14$ is read: 9 and 5 are 14, or 9 plus 5 equals 14. Beginners should adopt the first reading.

Again, $8 + 3 = ?$ is read: 8 and 3 are how many.

Exercises.

55. Read:

1. $8 + 5 = 13$.
2. $9 + 6 = 15$.
3. $5 + 4 = ?$
4. 6c. + 4c. = 10c.
5. \$7 + \$10 = \$17.
6. \$6 + \$3 = ?
7. 7 men + 4 men = 11 men.
8. 5 pins + 6 pins = ?

56. Express by signs:

1. 3 and 5 are 8.
2. 2 and 7 are what?
3. 3 + 4 equals what?
4. 3c. and 10c. are 13c.
5. 6 balls added to 3 balls are 9 balls.
6. How many are 3 and 12?
7. The sum of 6 and 2 is 8.
8. \$4 and \$8 are how many?

57. Principle.—*Only like numbers can be added.*

58. The Complemental Parts of a number are the numbers whose sum equals that number.

Thus: the complemental parts of 7 are 3 and 4, or 1, 2, 3 and 1.

59. By Addition we find a number when its complemental parts are given.

Complemental will frequently be denoted by the letter **c.**

Thus, the **c** parts of 8 are 5 and 3, or 6 and 2.

Suggestion to Teachers.—In all the examples in Addition, the pupil should be required to point out the **c** *parts* and the *whole.*

60. The following table should be thoroughly committed to memory:

Addition Table.

1.	2.	3.	4.	5.
1+1= 2	2+1= 3	3+1= 4	4+1= 5	5+1= 6
1+2= 3	2+2= 4	3+2= 5	4+2= 6	5+2= 7
1+3= 4	2+3= 5	3+3= 6	4+3= 7	5+3= 8
1+4= 5	2+4= 6	3+4= 7	4+4= 8	5+4= 9
1+5= 6	2+5= 7	3+5= 8	4+5= 9	5+5=10
1+6= 7	2+6= 8	3+6= 9	4+6=10	5+6=11
1+7= 8	2+7= 9	3+7=10	4+7=11	5+7=12
1+8= 9	2+8=10	3+8=11	4+8=12	5+8=13
1+9=10	2+9=11	3+9=12	4+9=13	5+9=14
6.	**7.**	**8.**	**9.**	**10.**
6+1= 7	7+1= 8	8+1= 9	9+1=10	10+1=11
6+2= 8	7+2= 9	8+2=10	9+2=11	10+2=12
6+3= 9	7+3=10	8+3=11	9+3=12	10+3=13
6+4=10	7+4=11	8+4=12	9+4=13	10+4=14
6+5=11	7+5=12	8+5=13	9+5=14	10+5=15
6+6=12	7+6=13	8+6=14	9+6=15	10+6=16
6+7=13	7+7=14	8+7=15	9+7=16	10+7=17
6+8=14	7+8=15	8+8=16	9+8=17	10+8=18
6+9=15	7+9=16	8+9=17	9+9=18	10+9=19

Note.—Pupils should read the tables thus: 1 and 1 are 2, 1 and 2 are 3, 2 and 1 are 3, 1 and 4 are 5, 4 and 1 are 5, etc. The table is expressed in signs to familiarize the pupil with their use and meaning.

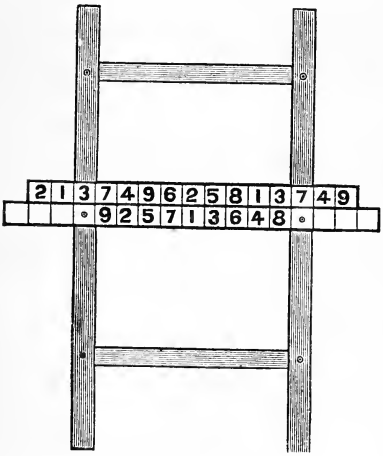
Drill Exercises.

61. The following, read vertically, are the two complemental parts of the number written under them:

1	2	32	43	543	654	7654	8765	98765
1	1	12	12	123	123	1234	1234	12345
2	3	4	5	6	7	8	9	10

9	9	98	98	987	987	9876	9876
9	8	78	67	567	456	3456	2345
18	17	16	15	14	13	12	11

Suggestions to Teachers.—These parts and their sum should be thoroughly committed to memory, so that they will be recognized at a glance. For this purpose the following is recommended: Procure two boards of convenient dimensions and dress them smoothly, so that one will slide freely on the other. Mark, number, and place them as shown in the diagram, and require the class in daily drill, to name the sum of the opposite numbers. Then shift the position of the upper board, and proceed as before, until all the combinations have been reached. These boards, especially with the addition of two or three similar ones, will afford abundant drill exercises in Addition, Subtraction, and Multiplication. We shall subsequently refer to them as the Combination Boards. See Art. 104.



Combination Boards.

Mental Exercises.

62. 1. Frank is 10 years old, and his sister is 3 years older; how old is his sister?

2. John has 6 pears, and William has 5 more than John; how many pears has William?

3. Susan has 7 roses and Lucy has 3 roses; how many roses have both girls?

4. Thomas killed 6 squirrels on Monday, and 8 squirrels on Friday; how many squirrels did he kill on both days?

5. 7 rabbits are in the garden, and 4 rabbits are in the yard; how many rabbits in all?

6. 1 hen has 8 chicks, and another hen has 5 chicks; how many chicks have both hens?

7. A boy worked 9 hours on Tuesday, and 6 hours on Wednesday; how many hours did he work on both days?

8. John caught 5 fishes, and Ben caught 8 fishes; how many were caught by both boys?

9. Moses is 9 years old now; how old will he be 7 years from now?

10. Emma has 8 tulips, and Ann has 8 tulips more than Emma; how many tulips has Ann?

11. A man traveled 6 miles, and then went 3 miles further; how far did he travel in all?

12. Henry spent 10 cents for oranges, and 6 cents for apples; how much did he spend in all?

13. How many are 2 and 2? 3 and 4? 4 and 5? 6 and 6?

14. 7 and 7? 8 and 8? 9 and 9? 5 and 6? 7 and 3? 8 and 4?

15. 9 and 2? 5 and 7? 6 and 8? 7 and 9? 2 and 5? 1 and 8?

16. 6 and 4? 5 and 5? 8 and 2? 5 and 3? 9 and 9? 6 and 7?

17. What is the number whose complemental parts are 2 and 3? 3 and 4? 4 and 5? 5 and 6? 9 and

4? 7 and 8? 1, 3, and 4? 2, 3, and 7? 3, 5, and 8? 1, 2, 3, and 4?

18. How many are 5 and 6? 15 and 6? 25 and 6? 35 and 6? 45 and 6? 55 and 6? 65 and 6? 75 and 6? 85 and 6? 95 and 6?

19. How many are 8 and 7? 18 and 7? 28 and 7? 38 and 7? 48 and 7? 58 and 7? 68 and 7? 78 and 7? 88 and 7? 98 and 7?

20. How many are 9 and 8? 19 and 8? 29 and 8? 39 and 8? etc., to 107.

21. How many are 4 and 9? 14 and 9? 24 and 9? etc., to 103.

22. How many are 7 and 5? 17 and 5? 27 and 5? etc., to 102.

23. How many are 4 and 10? 14 and 10? 24 and 10? etc., to 104.

24. How many are 4 and 5? 5 more than that? 5 more than that? 5 more than that? 5 more than that?

25. How many are 7 and 3? 3 more than that? 3 more than that? 3 more than that? etc., to 22.

26. How many are 8 and 4? 4 more than that? etc., to 32.

27. How many are 9 **a**'s and 6 **a**'s? 6 **a**'s more than that? etc., to 33 **a**'s.

28. How many are 6 **c**'s and 7 **c**'s? 7 **c**'s more than that? etc., to 41 **c**'s.

29. How many are 2 **n**'s and 8 **n**'s? 8 **n**'s more than that? etc., to 32 **n**'s.

30. How many are 3 cows and 6 cows? 7 tens and 4 tens? 9 fives and 3 fives? 5 thirds and 2 thirds? 8 tenths and 4 tenths?

Exercises in Making Problems.

63. 1. Make a problem of 5 **b** + 7 **b** = ?

Ans. 5 bats and 7 bats are how many?

Or, Mary has 5 beads and Susan has 7 beads; how many beads have both girls?

2. Make a problem of 6 **m** + 8 **m** = ?

Ans. 6 men and 8 men are how many?

Or, there are 6 marbles in one pile, and 8 marbles in another pile; how many marbles in both piles?

Make problems of the following, and then give the answers:

3. 7 **c** + 8 **c** = ?
4. 6 **f** + 9 **f** = ?
5. 10 **h** + 5 **h** = ?
6. 36 **a** + 5 **a** = ?
7. 54 **d** + 9 **d** = ?
8. 10 **p** + 6 **p** + 5 **p** = ?

Slate Exercises.

64. Copy and add the following, placing the answer under the line:

4	9	5	7	8	4	7	10	34	46	57
7	6	4	7	5	9	8	9	3	8	9

3	5	3	5	6	5	3	8	42	34	16
2	4	0	4	3	2	5	9	2	5	4
4	7	8	9	7	2	4	7	7	3	6

5	3	1	2	3	4	3	4	3	4	5
2	7	9	7	5	6	7	8	1	6	7
1	2	2	5	5	6	3	8	9	7	8
4	5	8	7	5	6	7	4	8	10	8

7	3	9	6	5	7	4	3	5
8	5	8	3	4	8	9	2	3
6	9	7	5	9	5	3	7	9
5	8	6	9	10	3	8	10	5
4	12	15	17	27	30	20	36	17

The following exercises may be performed, first on the slate, and then mentally:

Mental Exercises.

65. 1. Name every 10th number from: 0 to 100: 5 to 105; 2 to 102; 7 to 107; 9 to 109; 3 to 103; 8 to 108; 4 to 104; 1 to 101; 6 to 106.

2. Count 8 on to: 7; 15; 21; 34; 46; 52; 68; 72; 89; 97.

3. What is the sum of: 23 and 7? 37 and 5? 48 and 3? 59 and 4? 66 and 8? 72 and 9? 84 and 7? 99 and 3? 45 and 5? 76 and 7? 93 and 5? 87 and 8? 63 and 8?

4. $16+7=?$ $35+6=?$ $43+8=?$ $74+3=?$ $98+7=?$ $57+9=?$

5. To every 10th number from 5 to 95 add: 7; 3; 8.

Add or count:

6. By 2's from 0 to 18.
7. By 3's from 0 to 30.
8. By 4's from 0 to 40.
9. By 5's from 0 to 50.
10. By 6's from 0 to 60.
11. By 7's from 0 to 70.
12. By 8's from 0 to 80.
13. By 9's from 0 to 90.

Name:

14. Every 3d number from 2 to 32.
15. Every 4th number from 3 to 39.

16. Every 5th number from 4 to 104.
17. Every 6th number from 5 to 65.
18. Every 7th number from 6 to 76.
19. Every 8th number from 7 to 87.
20. Every 9th number from 8 to 71.
21. What pairs of digits, when added, will make: 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16?

Find the sum of:

22. 5, 7, 9, 3.
23. 2, 4, 6, 8.
24. 7, 1, 9, 3, 5.
25. 10, 6, 8, 4, 7.
26. \$7, \$8, \$9.
27. 6c, 7c, 3c, 9c.
28. 5 hats, 4 hats, 7 hats.
29. 16 m, 10 m, 9 m, 7 m.
30. \$50, \$10, \$9, \$7, \$3, \$5,
31. $6+8+3=?$
32. $9+5+6=?$
33. $10+6+7+3=?$
34. $23+7+6+5=?$
35. $7+3+9=?$
36. $9+7+8=?$
37. $34+10+6+5=?$
38. $45+9+7+3=?$
39. How many are 35 and 20?

Operation.—3 tens and 2 tens = 5 tens or 50. $50+5=55$.

40. There are 45 boys and 50 girls in school. How many pupils in all?
41. If Charles reads 78 pages one day and 60 pages the next day, how many pages will he read in both days?
42. A drover bought 67 sheep from one man and 30 from another, how many sheep did he buy of both?
43. How many are 63 and 20? 78 and 40? 37 and 70? 45 and 30? 97 and 40? 50 and 23? 70 and 85? 90 and 44? 80 and 56? 60 and 84?
44. How many are 49 and 37?

Operation.—$49+30+7=79+7=86$.

45. John gave 25 cents for a slate and 42 cents for a book; what did both cost?

46. A pole is 43 feet in the air, 19 feet in the earth, and 18 feet in the water. How long is the pole?

47. A lad, having spent 43 cents, finds he has 59 cents left. How much had he at first?

48. How many are 43 and 22? 64 and 53? 47 and 35? 84 and 21? 74 and 56? 73 and 91? 53 and 41? 75 and 92? 39 and 24? 43 and 87?

49. How many are 2 tens and 12 *ones?*

OPERATION.—12 ones=1 ten 2 ones. 2 tens and 1 ten 2 ones are 3 tens 2 ones = 32.

50. How many are 3 tens and 25 ones? *Ans.* 5 tens 5 ones = 55.

51. How many are: 4 tens and 17 ones? 6 tens and 34 ones? 8 tens and 73 ones?

WRITTEN EXERCISES.

66. 1. Find the sum of 4864, 785 and 693.

OPERATION.

$$\begin{array}{r} 4864 \\ 785 \\ 693 \\ \hline 6342 \end{array}$$

EXPLANATION.—Write the numbers or *parts*, so that units of the same order stand in the same column—*ones* under *ones*, *tens* under *tens* etc.; and draw a line beneath them.

Adding, from the bottom, the column on the right, we get 12 (=1 ten and 2 ones); write the two below the line for the ones of the required sum. Adding the 1 ten with the tens of the given parts, which is the next column, we get 24 (=2 *hundreds* and 4 *tens*); write the 4 below the line for the *tens* of the required sum. Adding the 2 hundreds to the hundreds of the given parts, which are the numbers of the 3d column, we get 23 (=2 *thousands* and 3 *hundreds*); write the three in the place of hundreds in the sum, and carry the 2 to the next column of thousands, which, added to 4, gives 6 thousands. Hence the sum is 6342.

To prove the work we begin at the top and add down.

2. $45364 + 8965 + 786 + 9374 + 47 = ?$

OPERATION.

45364
8965
786
9374
47
64536

67. In adding it is best to use only the following wording: 7, 11, 17, 22, 26 (emphasize 6, and write it down while pronouncing it), carry 2; 6, 13, 21, 27, 33, carry 3; 6, 13, 22, 25, carry 2; 11, 19, 24, carry 2; 6.

From the preceding examples we derive the following

RULE.—I. *Write the parts so that like orders of units shall stand under each other.*

II. *Begin at the right, add each column separately, write the units' figure of the sum under the column added, and carry the tens, if any, to the next column.*

PROOF.—*Perform the addition in the reverse direction, from top to bottom, and if the results agree the work is probably correct.*

In this manner add and prove:

(3)	(4)	(5)	(6)	(7)	(8)
87	49	64	37	65	48
96	70	89	98	34	84
Ans. 183					

(9)	(10)	(11)	(12)	(13)	(14)
95	21	52	78	80	67
36	54	76	89	63	27
48	79	81	67	94	57
Ans. 179					

(15)	(16)	(17)	(18)	(19)	(20)
939	818	729	608	590	486
827	706	695	587	405	705
705	694	581	446	967	628
693	582	434	993	879	530
Ans. 3164					

(21)	(22)	(23)	(24)	(25)
38025	75631	9998	642	43384
9467	8467	74635	9753	965
7098	983	672	85671	8741

Ans. 54590

68. Table of distances on the Mississippi River, compiled in integers of miles, from the surveys of the Mississippi River Commission:

Jetties to New Orleans . .	96	Vicks. to La. State Line . .	47
N. O. to Donaldsville, La. .	78	La. S. L. to Greenville, Miss.	44
Donald. to Plaquemine, La.	32	Green. to Arkansas City, Ark.	40
Plaq. to Baton Rouge, La. .	20	A. C. to mouth Ark. R., Ark.	37
B. R. to Bayou Sara, La. .	34	Mou. Ark. R. to Helena, Ark.	95
B. S. to Mouth Red R., La.	35	Helena to Memphis, Tenn.	76
Mouth R. R. to Natchez, Miss.	64	Memp. to Fort Pillow, Tenn.	58
Natchez to St. Joseph, La. .	52	Fort P. to Columbus, Ky. .	151
St. J. to Vicksburg, Miss. .	49	Columbus to Cairo, Ill. . .	21

How far is it from New Orleans by river:

26. To Baton Rouge? *Ans.* 130 miles.

27. To Natchez? *Ans.* 263 "

28. To Vicksburg? *Ans.* 364 "

29. To Memphis? *Ans.* 733 "

30. How far is it from the mouth of Red River to the mouth of Arkansas River? To the Jetties?

Ans. 363 miles; 295 miles.

31. How far will a man travel who takes a boat at Baton Rouge, La., and goes to Helena, Ark. *Ans.* ?

32. How far is it from the Jetties to Memphis? Cairo? *Ans.* ?

33. A farmer sold 4 bales of cotton; the first weighed 463 pounds, the second 458 pounds, the third 417 pounds, and the fourth 513 pounds; what was the whole weight? *Ans.* 1851 pounds.

34. An army is composed of 34379 infantry, 8625 cavalry, and 1792 artillery-men; how many men in the army? *Ans.* 44796 men.

35. A butcher bought 6 oxen which weighed 1345 pounds, 1623 pounds, 978 pounds, 1174 pounds, 819 pounds, and 1796 pounds; what was the total weight? *Ans.* ?

Find the sum of:

36. 2564, 34875, 16374, 985, 76. *Ans.* 54874.

37. 14200 yards, 672 yards, 1265 yards, 3789 yards. *Ans.* 19926 yards.

38. 340 acres, 281 acres, 57 acres, 426 acres, 5 acres. *Ans.* 1109 acres.

39. 25 days, 460 days, 191 days, 763 days, 1084 days. *Ans.* ?

NOTE.—When numbers of dollars and cents are to be added, they must be written so that the points stand under each other.

40. \$36.27 + \$5.96 + \$1208. + \$120.40 + \$75.00 + \$.94 = ? *Ans.* \$1446.57.

41. \$50.04 + \$7.80 + \$102.10 + \$15.08 + \$208.00 + \$3.43 = ? *Ans.* \$386.45.

42. \$304.00 + \$75.75 + \$12.05 + \$27.54 + \$5.81 + \$63.02 = ? *Ans.* ?

43. What is the amount of \$45.63, \$3.68, \$37.45, \$93.07, \$2.84, \$175.50, and \$430.12? *Ans.* \$788.29.

44. Mr. Hicks owes Mr. Johnson \$130.50, Mr. Jackson \$475.12, Mr. Turner \$980, and Mr. Wafer \$17.64; how much does he owe them all? *Ans.* \$1603.26.

45. A farmer sold a horse for \$109.50, four bales of cotton for \$197.85, three hogsheads of sugar for \$239, and a load of corn for \$13.25; what did he receive for all? *Ans.* ?

46. North America contains 8,593,000 square miles,

South America 7,362,000 square miles, Europe 3,825,000 square miles, Asia 17,300,000 square miles, and Africa 11,557,000 square miles; how many square miles are in these five countries? *Ans.* 48,637,000 square miles.

69. When the *parts* are equal, the *whole* or *sum* may be indicated by inclosing one of the equal parts in a parenthesis, and writing the number of parts before it.

Thus, 5 (7) indicates 5 *sevens*, or $7+7+7+7+7=35$.

Exercises.

1. 4 (357) = ?

OPERATION.

$$\begin{array}{r} 357 \\ 357 \\ 357 \\ 357 \\ \hline 1428 \end{array}$$

2. 5 (645) = ? *Ans.* 3225.
3. 7 (807) = ? *Ans.* 5649.
4. 3 (2768) = ? *Ans.* 8304.
5. 9 (215) = ? *Ans.* 1935.
6. 6 ($435.10) = ? *Ans.* ?
7. 7 (374 hogs) = ? *Ans.* ?
8. 4 (2568 lbs.) = ? *Ans.* ?
9. 10 (504 cents) = ? *Ans.* ?

10. A father gave each of his five sons $125; how much did he give them all? *Ans.* $625.

11. What is the total weight of 3 bales of cotton if each bale weighs 410 pounds? *Ans.* 1230 pounds.

12. James has 5 boxes, and in each box there are 175 chestnuts; how many chestnuts has he in all? *Ans.* ?

13. What will 6 horses cost if each horse cost $150? *Ans.* ?

70. Parallel Problems.

Note.—In these and in subsequent parallel problems the *mental*, denoted by **m**, involve the same principles as the succeeding written problem or problems, and are intended for two purposes, viz:

1°. To supply the place of mental arithmetic; hence, they should be solved mentally and recited orally.

2°. To furnish *indirectly* an explanation of the principles and terms embraced in the parallel written problems, so that the pupil, by proper diligence, may comprehend and solve the latter unaided.

1.ᵐ What is the 9th number above 12?

2. What is the 837th number above 968? *Ans.* 1805.

3.ᵐ I sold a hog for $13, which lacked $5 of being as much as the hog cost me; what did the hog cost?

4. By selling a farm for $1305 I lost $960.50; what did the farm cost me? *Ans.* $2265.50.

5.ᵐ James spent $8 for pants, $9 for a coat, and $5 for a vest; how much did he spend in all?

6. A merchant invests $3275 in a house, $9760.23 in merchandise, $2987.53 in improvements, and $1624.35 in clerks' hire; what is his total investment?

Ans. $17647.11.

7.ᵐ John gave his mother 12 pears, his father 7 pears, his sister 6 pears, and had 5 pears left; how many had he at first?

8. A farmer paid $4875.60 for a farm, $1782 for stock, $2416.98 for supplies, and had $3026.05 left; how much money had he at first? *Ans.* $12100.63.

9.ᵐ William paid $7 for a hog, $9 for a calf, $5 for a sheep, and sold them so as to make $6; what did he receive for all?

10. A speculator bought a drove of horses for $3764.15, a drove of cattle for $2017.55, a drove of sheep for $620, and sold them at a profit of $784.85; what did he receive for all? *Ans.* $7186.55.

11.ᵐ James has 3 marbles, John 5 more than James, and Moses 6 more than John; how many have all?

12. Mr. Taylor has 735 sheep, Mr. Jackson has 842

more than Mr. Taylor, and Mr. Hulse 634 more than Mr. Jackson; how many have the three men?

Ans. 4523 sheep.

13.ᵐ Six years ago Peter was 9 years old; how old will he be 8 years from now?

14. A man married 17 years since, at which time he was 24 years old; how old will he be 22 years hence?

Ans. 63 years.

15.ᵐ Add by 9's from 0 to 45.

16. Add or count by 278's from 0 to 1390.

Ans. 0, 278, 556, etc.

71. Questions for Review.

What are: 1. Like numbers? 2. Unlike numbers? What is Addition? What is the sign of Addition? What is denoted: 1. By the sign =? 2. By the sign (?)? What are the complemental parts of a number? What do we find by Addition? What stands for complemental?

What is the: 1. Principle of Addition? 2. Rule for Addition? How may we prove Addition?

What is meant by parallel problems? *Ans.* Problems which involve the same principles.

SUBTRACTION.

INDUCTIVE EXERCISES.

72. 1. Count 7 backward. *Ans.* 7, 6, 5, 4, 3, 2, 1.

2. Count $7 backward. *Ans.* $7, $6, $5, $4, $3, $2, $1.

3. How do we count backward?

4. In counting backward, what are the next three numbers below $7?

5. How many are left when $1 is taken 3 times from $7?

6. In counting backward, what are the next 4 numbers below 10 hats? *Ans.* 9 **h**'s, 8 **h**'s, 7 **h**'s, 6 **h**'s.

7. How many are left when 4 hats are taken from 10 hats? *Ans.* 6 hats.

In counting backward, what is:	How many are:
8. The 1st number below 9?	8. 9 less 1?
9. The 3d number below 12?	9. 12 less 3?
10. The 3d number below $8?	10. $8 less $3?
11. The 4th number below 12 **c**?	11. 12 **c**'s less 4 **c**'s?

12. If I make 7 marks ///////, and rub out 3 of them, how many will be left?

13. John had 12 balls and gave James 5; how many balls did John have left?

14. Eight peaches are on a tree, if 5 are taken off, how many will be left?

How many are:

15. \$7 less \$1?
16. 9 men less 2 men?
17. 8 pints less 3 pints?

Subtract:

15. \$1 from \$7.
16. 2 men from 9 men.
17. 3 pints from 8 pints.

Definitions.

73. Subtraction is the process of taking from a number a given number of like units.

74. The number to be diminished is called the **Minuend,** the number by which it is diminished, the **Subtrahend,** and the result the **difference** or **remainder.**

Thus: \$3 taken from \$7 leaves \$4. Here, \$7 is the minuend, \$3 the subtrahend, and \$4 the remainder.

75. The sign of Subtraction is —, which is read: *less* or *minus.* When — stands between two numbers it indicates that the one *after* it is to be taken from the one *before* it.

Thus: $8-3=5$ is read: 8 less 3 is 5, or 8 minus 3 equals 5. Beginners should adopt the first reading, or: 3 from 8 leaves 5.

Again, $10-4=?$ is read: 10 less 4 is how many? or 4 from 10 leaves how many?

Exercises.

76. Read:

1. 9 **m**'s — 6 **m**'s = 3 **m**'s.
2. 7 **d**'s — 4 **d**'s = 3 **d**'s.
3. 12 boys — 7 boys = ?
4. $6+5-3=8$.
5. $7+8-6-4=5$.
6. $10-6+5-3=6$.

77. Express by signs:

1. \$14 minus \$5 is \$9.
2. \$12 less \$7 is \$5.
3. 7 plus 5 minus 3 is 9.
4. 16 less 7 is how many?
5. 3 from 11 leaves 8.
6. 6 from 14 leaves 8.

In each of the preceding examples point out the minuend, subtrahend, and remainder. Thus, in Ex. 3, 7 plus 5 is the minuend, 3 is the subtrahend, and 9 the remainder.

78. PRINCIPLE.—*Only like numbers can be subtracted.*

RELATION OF SUBTRACTION TO ADDITION.

79. 4 and 3 are how many? 4 and *what* number are 7? *What* number and 3 are 7?

$4 + ? = 7$? *Ans.* 3; because $7 - 4 = 3$.

$? + 3 = 7$? *Ans.* 4; because $7 - 3 = 4$.

In a similar manner answer the following:

$9 + ? = 12$. $? + 7 = 15$. 5 cents $+ ? = 11$ cents.

$8 + ? = 14$. $? + 3 = 10$. $? + 6$ pecks $= 9$ pecks.

PRINCIPLES.

1°. Subtraction is the *reverse* of Addition.

2°. By Subtraction we find one of the complemental parts of a number, when the number and the other part are given.

Thus: if 6 is one of the **c** parts of 11, the other part is $11 - 6$, or 5.

SUGGESTIONS TO TEACHERS.—In all the examples and problems in Subtraction, the pupil should be required to point out the **c** *parts* and the *whole*.

Thus, in $11 - 5 = 6$, 5 and 6 are the parts, and 11 is the whole. Again, in the problem: James had \$15, but lost \$8, how much had he left? \$8 and \$7 are the parts, and \$15 the whole.

80. Since Subtraction is the reverse of Addition, by reversing the table of the latter, we get the

Subtraction Table.

1	2	3	4	5
1 — 1 = 0	2 — 2 = 0	3 — 3 = 0	4 — 4 = 0	5 — 5 = 0
2 — 1 = 1	3 — 2 = 1	4 — 3 = 1	5 — 4 = 1	6 — 5 = 1
3 — 1 = 2	4 — 2 = 2	5 — 3 = 2	6 — 4 = 2	7 — 5 = 2
4 — 1 = 3	5 — 2 = 3	6 — 3 = 3	7 — 4 = 3	8 — 5 = 3
5 — 1 = 4	6 — 2 = 4	7 — 3 = 4	8 — 4 = 4	9 — 5 = 4
6 — 1 = 5	7 — 2 = 5	8 — 3 = 5	9 — 4 = 5	10 — 5 = 5
7 — 1 = 6	8 — 2 = 6	9 — 3 = 6	10 — 4 = 6	11 — 5 = 6
8 — 1 = 7	9 — 2 = 7	10 — 3 = 7	11 — 4 = 7	12 — 5 = 7
9 — 1 = 8	10 — 2 = 8	11 — 3 = 8	12 — 4 = 8	13 — 5 = 8
10 — 1 = 9	11 — 2 = 9	12 — 3 = 9	13 — 4 = 9	14 — 5 = 9
6	**7**	**8**	**9**	**10**
6 — 6 = 0	7 — 7 = 0	8 — 8 = 0	9 — 9 = 0	10 — 10 = 0
7 — 6 = 1	8 — 7 = 1	9 — 8 = 1	10 — 9 = 1	11 — 10 = 1
8 — 6 = 2	9 — 7 = 2	10 — 8 = 2	11 — 9 = 2	12 — 10 = 2
9 — 6 = 3	10 — 7 = 3	11 — 8 = 3	12 — 9 = 3	13 — 10 = 3
10 — 6 = 4	11 — 7 = 4	12 — 8 = 4	13 — 9 = 4	14 — 10 = 4
11 — 6 = 5	12 — 7 = 5	13 — 8 = 5	14 — 9 = 5	15 — 10 = 5
12 — 6 = 6	13 — 7 = 6	14 — 8 = 6	15 — 9 = 6	16 — 10 = 6
13 — 6 = 7	14 — 7 = 7	15 — 8 = 7	16 — 9 = 7	17 — 10 = 7
14 — 6 = 8	15 — 7 = 8	16 — 8 = 8	17 — 9 = 8	18 — 10 = 8
15 — 6 = 9	16 — 7 = 9	17 — 8 = 9	18 — 9 = 9	19 — 10 = 9

Note.—Pupils should read the tables thus: 1 from 1 leaves none, 1 from 2 leaves 1, 1 from 3 leaves 2, etc. The table is expressed in signs to familiarize the pupil with their use and meaning.

Drill Exercises.

81. In these exercises each figure is to be subtracted from the number that stands above the group. The exercises should be written on the board, and used in class drill daily, until every pupil can call all the results instantly.

2	3	4	5	6	7	8
1	12	123	1324	13542	153246	1635437
18	17	16	15	14	13	12
9	89	798	6897	58796	486597	3856479

9	10	11
18347562	183659742	63752849

Mental Exercises.

82. 1. Susan is 14 years old, and her brother is 5 years younger; how old is her brother?

2. William has 11 pears, and John has 6 less than William; how many pears has John?

3. Susan and Lucy together have 10 roses; if 7 of them are Susan's, how many has Lucy?

4. Thomas killed 8 sqirrels on Tuesday, and 14 on Tuesday and Wednesday together; how many did he kill on Wednesday?

5. There are 13 rabbits in the garden and yard; if 6 rabbits are in the yard, how many are in the garden?

6. Two hens have 13 chicks together; if one hen has 8 chicks, how many chicks has the other?

7. A boy worked 15 hours in two days; if he worked 6 hours in one day, how many did he work the other?

8. Ben caught 17 fishes and Moses 9 fishes; how many more fishes did Ben catch than Moses?

9. 7 years from now Henry will be 15 years old; how old is he now?

10. Emma has 14 tulips and has 7 tulips more than Ann; how many tulips has Ann?

11. A man traveled 9 miles; how far would he have traveled if he had gone 5 miles less?

12. Harry spent 16 cents for oranges and 8 cents less for apples; how much did he spend for apples?

13. How many is: 13 less 6? 17 less 8? 8 less 5?

14. 10 less 2? 10 less 5? 9 less 8? 7 less 5? 16 less 8?

15. 16 less 7? 14 less 6? 12 less 7? 11 less 9? 10 less 6?

16. 10 less 3? 11 less 5? 18 less 9? 16 less 9? 14 less 7?

17. If 7 is one of the **c** parts of 15, what is the other?

18. If 9 is one of the **c** parts of 13, what is the other?

19. How many is: 104 less 6? 94 less 6? 84 less 6? 74 less 6? 64 less 6? 54 less 6? 44 less 6? 34 less 6? 24 less 6? 14 less 6?

20. How many is: 103 less 7? 93 less 7? 83 less 7? 73 less 7? 63 less 7? 53 less 7? 43 less 7? 33 less 7? 23 less 7? 13 less 7?

21. How many is: 105 less 9? 95 less 9? etc., to 6.

22. How many is: 101 less 10? 91 less 10? etc., to 1.

23. How many is: 53 less 8? 8 less than that? 8 less than that? 8 less than that? 8 less than that?

24. How many is: 43 less 5? 5 less than that? etc., to 3.

25. How many is: 65 less 9? 9 less than that? etc., to 2?

26. How many is 33 diminished by 7 four times?

27. How many is 30 less 5, less 5, less 5, less 6?

28. 17 **a**'s — 9 **a**'s = ? 25 **b**'s — 6 **b**'s = ? 32 **c**'s — 5 **c**'s = ? 40 **n**'s — 10 n's = ?

Exercises in Making Problems.

83. **1.** Make a problem of: 11 **s** — 5 **s** = ?

Ans. 5 spoons from 11 spoons leaves how many?

Or, James had 11 strings, but gave 5 strings to his sister; how many did he have left?

2. Make a problem of: 13 **y** — 6 **y** = ? *Problem:* Mike is 13 years old, and Henry is 6 years old; how much older is Mike than Henry?

Make problems of the following, giving the answers to each:

3. 9 **c** — 2 **c** = ?
4. 11 **r** — 8 **r** = ?
5. 17 **b** — 9 **b** = ?
6. 23 **a** — 6 **a** = ?
7. 32 **b** — 7 **b** = ?
8. 65 **m** — 9 **m** = ?

9. 7 **a** + 6 **a** — 5 **a** = ? *Ans.* If from the sum of 7 apples and 6 apples I take 5 apples, how many apples will be left?

10. 10 **b** + 6 **b** — 3 **b** = ?
11. 12 **a** + 9 **c** — 6 **c** = ?
12. 24 **g** + 8 **g** + 3 **g** — 6 **g** = ?
13. 15 **m** + 6 **m** — 4 **m** — 5 **m** = ?

Mental Exercises.

84. 1. Name every tenth number from: 100 to 0; 105 to 5; 102 to 2; 107 to 7; 109 to 9; 103 to 3; 108 to 8; 104 to 4; 101 to 1; 106 to 6.

2. Take 8 from: 15; 23; 29; 42; 54; 61; 76; 80; 97; 105.

3. What is the difference between 23 and 7? 42 and 5? 51 and 3? 63 and 4? 74 and 8? 81 and 9? 91 and 7? 102 and 3? 50 and 5? 83 and 7? 98 and 5? 95 and 8?

4. 23 — 7 = ? 41 — 6 = ? 51 — 8 = ? 77 — 3 = ? 105 — 7 = ? 66 — 9 = ?

5. To every 10th number from 105 to 5 subtract 7; 3; 8.

Subtract:

6. By 2's from 18 to 0.
7. By 3's from 30 to 0.
8. By 4's from 40 to 0.
9. By 5's from 50 to 0.
10. By 6's from 60 to 0.
11. By 7's from 70 to 0.
12. By 8's from 80 to 0.
13. By 9's from 90 to 0.

Name:

14. Every 3d number from 32 to 2.
15. Every 4th number from 39 to 3.
16. Every 5th number from 104 to 4.
17. Every 6th number from 65 to 5.
18. Every 7th number from 76 to 6.
19. Every 8th number from 87 to 7.
20. Every 9th number from 71 to 8.

21. $5 + 7 - 4 = ?$
22. $9 + 6 - 5 = ?$
23. $7 + 8 + 6 - 4 = ?$
24. $9 + 7 - 3 - 2 = ?$
25. $\$5 + \$8 - \$4 = ?$
26. $\$9 + \$7 + \$6 - \$3 = ?$
27. $10c. - 4c. + 7c. - 2c. = ?$
28. $40 - 12 - 8 - 5 = ?$

29. A man bought a cow for $30, and paid all but $10; how much did he pay?

30. Henry had 50 cents, but he paid 20 cents for some paper; how many cents had he left?

31. 40 gallons of syrup have been drawn from a tank that contained 90 gallons; how many gallons are left?

32. James has caught 60 fishes and Moses 40; how many more fishes must Moses catch to have as many as James?

33. Emma has 80 beads and Susan has 20 less than Emma; how many beads has Susan?

34. A man owing $70 gave his note for $15, and paid the balance in cash; how much cash did he pay?

35. I gave a horse worth $80 for a cow and $18 in money; how much did the cow cost me?

36. William had 42 oranges and gave 9 to Mary and 7 to James; how many oranges had he left?

37. Edward had $65, and after spending $15 lost all the balance except $12; how much did he lose?

38. From 64 take 27.

OPERATION: $64 - 20 = 44$, $44 - 7 = 37$.

39. From 75 take 36.
40. From 42 take 17.
41. From 83 take 65.
42. From 62 take 21.

43. $54 - 28 = ?$
44. $49 - 36 = ?$
45. $91 - 74 = ?$
46. $107 - 39 = ?$

WRITTEN EXERCISES.

85. CASE I.—When each figure of the subtrahend is less than the corresponding figure of the minuend.

1. From 695 subtract 324.

EXPLANATION.—695 = 6 hunds. 9 tens 5 ones.
324 = 3 hunds. 2 tens 4 ones.
Subtracting like numbers, we have 3 hunds. tens 1 one = 371.

OPERATION.

695
324
371

Hence, the

RULE.—I. *Write the subtrahend under the minuend, placing units under units, tens under tens, etc.*

II. *Begin at the right, subtract each figure of the subtrahend from the figure above it, and write the difference below.*

From:

2. 375 take 124. *Ans.* 251.
3. 687 take 505. *Ans.* 182.
4. 983 take 823. *Ans.* 160.
5. 841 take 31. *Ans.* 810.
6. 9324 take 113. *Ans.* 9211.
7. 7379 take 7163. *Ans.* 216.
8. 23946 take 1814. *Ans.* 22132.
9. 68438 take 225. *Ans.* 68213.
10. 79634 take 8420. *Ans.* 71214.

11. John had 84 apples and gave his mother 51 of them; how many had he left? *Ans.* 33 apples.

12. A farm contains 649 acres; if 332 acres should be sold, how many acres would be left? *Ans.* ?

13. Henry gathered 4845 chestnuts and gave 2104 of them to his sister; how many had he left? *Ans.* 2741.

14. A farmer has 679 sheep in one field, and 420 sheep in another field. How many more sheep in one field than the other? *Ans.* ?

15. How many more are 872 gallons than 501 gallons? *Ans.* 371 gallons.

16. One of the parts of 666 is 234; what is the **c** part? *Ans.* 432.

86. CASE II.—When any figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 764 take 481.

EXPLANATION.—Beginning at the right, we proceed as before, and say 1 from 4 leaves 3; since 8 *tens* is more than 6 tens, we take 1 hund. from the 7 hunds., add it to the 6 tens, making 16 tens. Now, 8 tens from 16 tens leave 8 tens, and since we took 1 from 7, we say 4 from 6 leaves 2.

OPERATION.

764
481
283

Hence,

RULE.—I. *Proceed as far as possible as in Case I.*

II. *When any figure of the subtrahend is greater than the one above it, add* 10 *to the latter and subtract, then diminish by* 1 *the units of the next higher order in the minuend.*

PROOF.—*Add the remainder and the subtrahend; their sum should be the minuend.*

87. Instead of *diminishing* by 1 the units of the next higher order in the *minuend*, it is more convenient in practice to increase by 1 the units of the next higher

order in the subtrahend. This is illustrated in the next example.

3. From 95283 take 8365.

Subtract thus: 5 from 13 leaves 8; carry 1 to 6 makes 7, 7 from 8 leaves 1; 3 from 12 leaves 9; carry 1 to 8 makes 9, 9 from 15 leaves 6; carry 1, 1 from 9 leaves 8.

OPERATION.

95283
8365
86918

Copy, subtract, and prove:

(4)	(5)	(6)	(7)
76	137	4672	6875
38	96	3708	928
Ans. 38	*Ans.* 41	*Ans.* 964	*Ans.* 5947

(8)	(9)	(10)	(11)
701	591	4703	32534
637	265	3928	18276
Ans. ?	*Ans.* ?	*Ans.* ?	*Ans.* ?

(12)	(13)	(14)	(15)
$520	$32.00	1371 days	70000 pins.
$361	$18.25	2859 days	36527 pins.
Ans. $159	*Ans.* ?	*Ans.* ?	*Ans.* ?

Subtract:

16. 99 from 125. *Ans.* 26.
17. 97 from 185. *Ans.* 88.
18. 961 from 1728. *Ans.* 767.
19. 965 from 2873. *Ans.* 1908.
20. 9283 from 11678. *Ans.* 2395.
21. 5398 from 83119. *Ans.* ?

What is the difference between:

22. 5873 dollars and 7325 dollars? *Ans.* $1452.
23. 158701 gallons and 98731 gallons? *Ans.* ?
24. 1158 sheep — 736 sheep = ? *Ans.* 422 sheep.
25. 7003 rocks — 2635 rocks = ? *Ans.* 4368 rocks.

How many more are:

26. 5000 cows than 3001 cows? *Ans.* 1999 cows.
27. 2375 threes than 1725 threes? *Ans.* 650 threes.
28. 83201 units than 64736 units? *Ans.* 18465 units.
29. 61111 thirds than 51115 thirds? *Ans.* 9996 thirds.
30. 7123 **a** than 6234 **a**? *Ans.* 889 **a**.

88. A list of a few celebrated mathematicians; their nativities, inventions, and the times of their births and deaths.

NAME.	NATIVITY.	INVENTIONS.	BIRTH.	DEATH.
Bowditch	American	Navigation Tables	1773	1838
Briggs	English	Common Logarithms	1556	1630
Cardan	Italian	Solution of Cubic Equat'ns	1501	1576
Demoivre	French	Trigonometrical Formulas	1667	1754
Descartes	French	Analytical Geometry	1596	1650
Euclid	Greek	Geometry	B. C. 300	——
Hamilton	Irish	Science of Quaternions	1805	1865
Lagrange	French	Calculus of Variations	1736	1813
Leibnitz	German	Dif. and Integral Calculus	1646	1716
Monge	French	Descriptive Geometry	1746	1818
Napier	Scotch	Logarithms	1550	1617
Newton	English	Binomial Theor., Calculus	1642	1727
Sturm	Swiss	Position of Real Roots	1803	1855
Taylor	English	Cal. of Finite Differences	1685	1731

How old was:

31. Cardan when Napier was born? *Ans.* 49 years.
32. Napier when Descartes was born? *Ans.* 46 years.
33. Descartes when Newton was born? *Ans.* 46 years.
34. Newton when Demoivre was born? *Ans.* 25 years.
35. Demoivre when Lagrange was born? *Ans.* 69 years.
36. Lagrange when Bowditch was born? *Ans.* 37 years.

37. How old was Bowditch when Hamilton was born?

Ans. 32 years.

38. At what age did Bowditch die? Monge? Newton? Taylor? Briggs? Leibnitz? Sturm?

39. How many years had Napier been dead when Newton was born?

40. A man had \$625 and spent \$235.75; how many dollars did he have left?

EXPLANATION.—We write the amounts so that the *points* stand under each other, annex two 0's to the minuend to supply the vacant *places* of cents, and subtract as in simple numbers.

OPERATION.

625.00
235.75
389.25

41. A man bought a horse for \$187 and a buggy for \$118.35; how much more did the horse cost than the buggy? *Ans.* \$68.65.

42. A farmer bought a wagon for \$113, and gave in exchange a cow worth \$43.75, and the balance in cash; how much was the balance? *Ans.* \$69.25.

43. Mount Sorata, a peak of the Andes, is 21,286 feet high, which is 5,506 feet higher than Mount Blanc, the highest peak of the Alps; how high is Mount Blanc?

Ans. 15780 feet.

44. In 1840 there were 2428921 inhabitants in the State of New York, and 1724033 inhabitants in the State of Pennsylvania; how many more inhabitants were there in New York than in Pennsylvania?

Ans. 704888.

45. In 1880 the number of male persons in Louisiana was 468233, and the number of females 471271; how many more females than males were there? *Ans.* 3038.

46. In 1840 the population of the U. S. was 17069453, and in 1880 it was 50155783; what was the increase in 40 years? *Ans.* 33086330.

89. 47. Three of the parts of 1250 are 231, 365, and 189; what is the **c** part?

EXPLANATION.—Since the **c** part added to the sum of the other parts make the *whole* 1250, by subtracting the sum of the given parts from the whole we get the **c** part. Hence, to find the **c** part when the whole and several parts are known, *add the given parts and subtract their sum from the whole.*

OPERATION.

231
365 1250
189 785
785 465 *Ans.*

48. Two of the parts of 35 are 12 and 15; what is the **c** part?

49. Two of the parts of \$980 are \$435 and \$386; what is the **c** part? *Ans.* \$159.

50. Three of the parts of \$1000 are \$555, \$222, \$111; what is the **c** part? *Ans.* \$112.

51. The sum of three numbers is 1160; the first number is 384, the second 571, what is the third? *Ans.* ?

52. A boy gathered 769 nuts, of which he gave his sister 263 and his mother 378; how many nuts had he left? *Ans.* 128.

53. A man starts on a journey of 583 miles; after he travels 260 miles, and then 173 miles more, and then 95 miles more; how far will he have to go?

Ans. 55 miles.

54. A man owed a debt of \$2000; at one time he paid \$520, at another \$763, and at another \$391; how much does he still owe? *Ans.* ?

90. Make problems of the following, giving the answer to each:

55. 33296 **c** − 22535 **c** = ?

56. 50000 **g** − 41715 **g** = ?

57. 6760 **d** − 3243 **d** − 2189 **d** = ?

58 8713 **b** + 1565 **b** − 3767 **b** − 4593 **b** = ?

91. Parallel Problems.

1.m One of the parts of 25 is 12; what is the **c** part?

2. A and B have together \$308, of which A owns \$183.25; how many dollars has B? *Ans.* \$124.75.

3.m Two of the parts of 33 are 11 and 9; what is the **c** part?

4. A farmer raised 3750 bushels of wheat, corn and barley, of which 1521 bushels were wheat, and 1038 bushels corn; how many bushels of barley did he raise? *Ans.* 1191.

5.m Jane had 45 peaches, of which she gave 20 to Emma, 10 to Rosa and 7 to Julia; how many did Jane have left?

6. A man having \$1768 on deposit, gave a check for \$175 to A, one for \$238.25 to B, and one for \$369.50 to C; how much money was left on deposit? *Ans.* \$985.25.

7.m James, William and Henry have together 37 oranges, of which 18 belong to James, and Henry has 9 oranges less than James; how many oranges has William?

8. The sum of three numbers is 6435; the first is 2816, and the second is 934 less than the first; what is the third? *Ans.* 1737.

9.m The **c** parts of a number are 12 and 13; what is the number?

10. The subtrahend is \$425.15, and the remainder \$172.85; what is the minuend? *Ans.* \$598.

11.m How much more is $18 + 12$ than $25 - 6$?

12. From the *sum* of 783 and 248 subtract the *difference* between 900 and 527. *Ans.* 658.

13.m What number added to 17 will make 30?

14. One of the parts of 8305 is 6971; what is the **c** part? *Ans.* 1334.

15.[m] One of the parts of $20 + 15$ is $10 + 5$; what is the **c** part?

16. A farmer received \$2025 for his sugar and \$1824 for his cotton. The expense of raising the sugar was \$1113, and of the cotton \$749; what were his profits?

Ans. \$1987.

17.[m] \$13 and \$8 are two parts of \$33; what is the **c** part?

18. A man paid \$5270 for a house and \$1835 for improving it. If he sells the house for \$7500, what will be his profits? *Ans.* \$395.

19.[m] If I begin at 40 and count backward, what will be the 12th number?

20. What is the 525th number below 937? *Ans.* 412.

21.[m] 50 contains 19, 11, and the **c** part; what is the latter?

22. A and B are 1529 feet apart; if A goes toward B 375 feet, and B goes toward A 682 feet, how far apart will they then be? *Ans.* 472 feet.

92. Questions for Review.

What is: 1. Subtraction? 2. The minuend? 3. The subtrahend? 4. The remainder? 5. The sign of Subtraction?

What is denoted by the sign $-$?

What is the relation of Subtraction to Addition? What do we find by Subtraction?

What is the: 1. Principle of Subtraction? 2. Rule for Subtraction? How may Subtraction be proved?

When we know the *whole* and several of its parts, how do we find the **c** part?

MULTIPLICATION.

INDUCTIVE EXERCISES.

93. 1. How many *ones* in 4 twos?

Ans. Two taken 4 times, or $2 + 2 + 2 + 2 = 8$.

2. How many are 5 *threes?*

Ans. Three taken 5 times, or $3 + 3 + 3 + 3 + 3 = 15$.

3. How many are 4 *sevens?*

Ans. The sum of four sevens, or 28.

4. How many are:

6 (5's)? 3 (9's)? 7 (4's)? 8 (5 dollars)? 5 (10 c.)? 9 (10's)?

5. One boy has two hands; how many hands have 6 boys? *Ans.* 2 hands taken 6 times, or 12 hands.

6. A horse has 6 nails in each of his 4 shoes; how many nails in all?

Ans. 4 (6 nails), or 6 nails taken as many times as there are shoes.

7. What will 5 hats cost at \$4 a piece? *Ans.* 5 (\$4).

8. At 9c. each, what will three melons cost?

9. What will be the cost of 6 pairs of boots at \$8 a pair? *Ans.* 6 (\$8) = ?

10. If a horse travels 4 miles per hour, how far does he travel in 7 hours?

11. There are 5 trees in the orchard and 20 peaches on each tree; how many peaches are in the orchard? 20 peaches are taken how many times? 5 (20 peaches) = ?

Definitions.

94. Multiplication is a short method of adding equal parts, or the process of taking one number as many times as there are units in another.

95. The number to be taken is called the **multiplicand;** the number that shows how many times it is to be taken, the **multiplier,** and the result, the **product.**

96. The *multiplicand* and *multiplier* are called the **factors** of the product.

97. The Sign of Multiplication is $\times$, which is read: *times*, or multiplied by. When $\times$ stands between two numbers, it indicates that one of them is to be multiplied by the other.

Thus, $6 \times 5 = 30$ is read: 5 times 6 are 30, or 6 multiplied by $5 = 30$. Here 6 is the multiplicand, 5 the multiplier, and 30 the product. The factors of 30 are 5 and 6.

Since $6 \times 5 = 5 \times 6$, either may be read: 5 times 6 or 6 times 5. If one of the factors is a concrete number, it is regarded as the multiplicand, but may be used abstractly as the multiplier. It is evident that 5 (6) also indicates multiplication. See Art. **69.**

98. The Complemental Factors of a number are those factors whose product is equal to that number.

Thus, the complemental factors of 6 are 2 and 3; of 12, 3 and 4, or 2 and 6; of 36, 4 and 9, or 2, 3, and 6, etc., etc.

99. Copy and read:

1. $4 \times 3 = 12$.
2. $5 \times 8 = 40$.
3. $9 \times 4 = ?$
4. $2 \times 9 = ?$
5. 6×2 yards $= 12$ yards.
6. 7×3 men $= 21$ men.
7. 8×7 boys $= 56$ boys.
8. 3×8 weeks $= ?$

Point out the *multiplicand*, *multiplier*, and *product* in each.

100. Express by signs:

1. 5 times 7 are 35.
2. 8 times 9 are 72.
3. 7 times $4 are $28.
4. 4 taken 3 times equals 12.
5. 9 times 5 are how many?
6. 6 times 4 pens are 24 pens.

101. PRINCIPLES.

1°. The *multiplier* is an *abstract* number.

2°. The *multiplicand* and *product* are *like* numbers.

3°. Multiplication may be effected by Addition.

Let the pupil point out or verify each of these principles in examples 1, 2, 5, 6, and 7 of Art. **99**.

Thus, in ex. 5,

1°. 6, the *multiplier*, is an *abstract* number.

2°. 2 yards, the *multiplicand*, and 12 yards, the *product*, are *like* numbers.

3°. 6×2 yards $= 2$ yards $+ 2$ yards $+ 2$ yards $+ 2$ yards $+ 2$ yards $+ 2$ yards $= 12$ yards.

102. By Multiplication we find a number when we know its complemental factors.

Thus, if 5 and 7 are the **c** factors of a number, the number is $5 \times 7 = 35$; if 6 and 9 are the **c** factors, the number is $6 \times 9 = 54$.

SUGGESTIONS TO TEACHERS.—In every example and problem in Multiplication the pupil should be required to point out:

1°. The whole and the **c** parts.

Thus, in $4 \times \$5 = ?$, the parts are $5, $5, $5, $5, and the whole is the answer required, viz: $20.

2°. The *whole* and the **c** factors.

Thus, in $6 \times 5 = ?$, the **c** factors are 6 and 5, and the whole is the answer required, 30.

103. The following table contains the products of each two numbers from 0 to 12. It should be thoroughly committed to memory.

Multiplication Table.

1	2	3	4
1 × 0 = 0	2 × 0 = 0	3 × 0 = 0	4 × 0 = 0
1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48

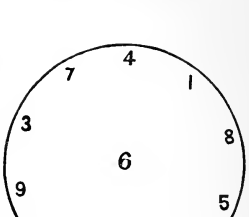
5	6	7	8
5 × 0 = 0	6 × 0 = 0	7 × 0 = 0	8 × 0 = 0
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96

9	10	11	12
9 × 0 = 0	10 × 0 = 0	11 × 0 = 0	12 × 0 = 0
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

Drill Exercises.

104. With the Combination Boards, Art. **61,** the teacher may provide abundant drill exercises in Multiplication. In the absence of these the following is recommended:

Draw a circle on the board, and within it write the figures from 0 to 9 inclusive, as in the diagram. At the center write 6, and let the pupils name the product of 6 by each of the other figures taken in order around the circle. Then erase 6, and in its place write one of the other figures to be used as a multiplier; and so continue until all the combinations shall have been reached.



This exercise should be used in class drill until every student can name all the products instantly. This circle may be used with equal facility in drilling pupils in Addition.

105. The following examples should be solved by Addition and Multiplication until the relation of the operations is clearly apprehended.

1. Multiply 9 by 7; 8 yards by 5.
2. What is the product of: 8 and 6? 5 and 7?
3. $7 \times 7 = ?$ $3 \times 8 = ?$ $6 \times 9 = ?$
4. What is the number whose **c** factors are 9 and 8?
5. What is the value of 6 if its unit is \$7? $6 \times 7 = ?$
6. What will 8 hats cost at \$5 apiece?

Mental Exercises.

106. What is the product of:

4 and 2? 5 and 3? 3 and 4? 6 and 2? 5 and 6?
4 and 7? 5 and 4? 4 and 5? 3 and 6? 5 and 7?
4 and 8? 6 and 7? 4 and 6? 7 and 2?

2. Multiply 3 by 3; 7 by 7; 6 by 6; 8 by 8; 5 by 5; 9 by 9; 4 by 4; 2 by 2; 6 by 8; 2 by 5; 6 by 3; 7 by 9; 9 by 3; 8 by 2.

3. $5 \times 9 = ?$ $2 \times 12 = ?$ $7 \times 8 = ?$ $3 \times 7 = ?$ $6 \times 4 = ?$ $9 \times 2 = ?$ $4 \times 11 = ?$ $6 \times 9 = ?$ $2 \times 10 = ?$ $4 \times 9 = ?$ $5 \times 11 = ?$

4. What is the number whose **c** factors are 3 and 4? 7 and 10? 9 and 8? 7 and 11? 3 and 8? 4 and 12? 8 and 11? 5 and 12? 3 and 2? 8 and 12? 9 and 11? 8 and 10?

5. What pairs of digits, when multiplied, will give the product: 12? 16? 18? 24? 36?

6. What is the value of 7 if its unit is 9? 7 *nines* are how many?

7. What is the value of 4 if its unit is 8? 4 *eights* are how many?

8. What is the value of 6 if its unit is: 3c? \$5? 7 hats? 4 days? 9? 2 feet? 6? 8 horses?

9.

$5 \times 4 + 10 = ?$	$3 \times 6 + 13 = ?$	$7 \times 2 - 5 = ?$
$4 \times 2 + 8 = ?$	$5 \times 7 - 12 = ?$	$3 \times 3 + 4 = ?$
$5 \times 6 - 10 = ?$	$4 \times 8 + 11 = ?$	$6 \times 6 - 3 = ?$
$4 \times 7 - 5 = ?$	$6 \times 7 - 10 = ?$	$7 \times 7 + 2 = ?$
$5 \times 4 + 12 = ?$	$4 \times 6 + 9 = ?$	$9 \times 9 - 1 = ?$
$5 \times 0 + 3 = ?$	$7 \times 0 + 0 = ?$	$8 \times 2 + 0 = ?$

NOTE.—Perform the multiplication first. Thus, $5 \times 2 + 3 = 10 + 3 = 13$.

EXERCISES IN MAKING PROBLEMS.

107. 1. Make a problem of: 5 **b** $\times 3 = ?$

Ans. 5 books taken 3 times are how many?

Or, If 1 barrel holds 5 bushels, how many bushels will 3 barrels hold?

Make a problem of:

2. $2\ \mathbf{c} \times 8 = ?$ *Ans.* ?
3. $7\ \mathbf{h} \times 4 = ?$ *Ans.* ?
4. $6\ \mathbf{d} \times 5 = ?$ *Ans.* ?
5. $9\ \mathbf{t} \times 6 = ?$ *Ans.* ?

6. Make a problem of: $6\ \mathbf{c} \times 4 + 5\ \mathbf{c} = ?$

Ans. How many are 4 times 6 cups, and 5 cups?

Or, 6 cups taken 4 times and 5 cups more are how many?

Make problems of the following, giving answers to each:

7. $3\ \mathbf{g} \times 8 + 3\ \mathbf{g} = ?$
8. $5\ \mathbf{a} \times 7 + 10\ \mathbf{a} = ?$
9. $6\ \mathbf{s} \times 7 - 5\ \mathbf{s} = ?$
10. $9\ \mathbf{b} \times 8 - 3\ \mathbf{b} = ?$

Mental Exercises.

108. 1. Of what number are 7 and 5 the **c** factors?

2. What is the cost of 8 barrels of flour at $7 a barrel?

Analysis.—Since 1 barrel cost $7, 8 barrels will cost 8 times $7, or $56.

3. One bushel contains 4 pecks; how many pecks in 7 bushels?

4. Seven days in a week, how many days in 8 weeks?

5. What is the cost of 9 ploughs at $11 each?

6. What is the cost of 12 hats at $5 each?

7. A horse walks 5 miles an hour, how far does he travel in 7 hours?

8. Ten cents are a dime and 10 dimes are a dollar; how many cents are in a dollar?

9. Three feet are a yard and 12 inches a foot; how many inches in a yard?

10. Seven days in a week and 4 weeks in a month, how many days in a month?

11. Forty rods in a rood and 4 roods in an acre, how many rods in an acre?

12. Four farthings in a penny and 12 pence in a shilling; how many farthings in a shilling?

13. What will be the cost of 4 barrels of flour at $10 a barrel?

14. At $9 each, what will be the cost of: 5 sheep? 7 sheep? 10 sheep? 6 hogs? 4 guns?

15. How many are 5×10? *Ans.* 5 *tens*, or 50.

16. How many are 6×10? *Ans.* 6 *tens*, or 60.

NOTE 1.—A number is multiplied by 10 by annexing one 0 to it.

17. How much will 10 cigars cost at: 5 cents apiece? 12 cents apiece? 15 cents apiece? 25 cents apiece?

18. How much will 16 lbs. sugar cost at 10 cents a pound?

19. There are 10 dimes in $1, how many dimes in: $3? $8? $27? $54? $120? $255?

20. There are 10 square chains in 1 acre; how many square chains in: 7 acres? 13 acres? 42 acres? 145 acres?

21. At $10 apiece, what will be the cost of: 9 hogs? 15 saddles? 21 guns? 32 clocks? 130 sheep?

22. How many are 6×100? *Ans.* 6 *hunds.*, or 600.

NOTE 2.—A number is multiplied by 100 by annexing two 0's to it.

23. What will 100 cigars cost at 4 cents apiece?

24. There are 100 cents in $1; how many cents in: $2? $19?

25. There are 100 years in 1 century; how many years in: 5 centuries? 11 centuries? 37 centuries?

26. At $100 apiece, what will be the cost of: 13 horses? 26 carriages? 30 organs? 143 gold watches?

27. How many are 7×1000? *Ans.* 7 *thous.*, or 7000.

28. How many are 7×10000?

Ans. 7 *ten-thous.* or 70000.

109. RULE. — *To multiply by* 10, 100, 1000, *annex as many* 0's *to the multiplicand as there are* 0's *in the multiplier.*

29. There are 1000 mills in 1 dollar; how many mills in \$2? \$3? \$9? \$12? \$63?

30. There are 1000 ounces in 1 cubic foot of water; how many ounces in 3 cubic feet of water? 8 cubic feet? 137 cubic feet?

WRITTEN EXERCISES.

110. CASE I.—When the multiplier is a digit.

1. Multiply 435 by 7.

OPERATION.

435
7
35
210
2800
3045

EXPLANATION.—Write the multiplier under the multiplicand, as in the margin, and beginning with the units, we say:

7×5 ones $= 35$ ones $= 35$,
7×3 tens $= 21$ tens $= 210$,
7×4 hunds. $= 28$ hunds. $= 2800$,

which added together make 3045.

In practice, the work is abbreviated thus: $7 \times 5 = 35$, write the 5 and carry 3; $7 \times 3 = 21$ and 3 make 24, write the 4 and carry 2; $7 \times 4 = 28$ and 2 make 30, which write.

OPERATION.

435
7
3045

Multiply:

2. 325 by 4.	*Ans.* 1300.	**5.** 487 by 3.	*Ans.* 1461.
3. 647 by 8.	*Ans.* 5176.	**6.** 792 by 6.	*Ans.* 4752.
4. 605 by 7.	*Ans.* 4235.	**7.** 986 by 5.	*Ans.* 4930.

8. In a similar manner solve the examples in Art. **69.**

9. Multiplicand is 643, multiplier is 7, product = ?

10. Multiplicand is $725, multiplier is 4, product = ?

11. Multiplicand is 6289c., multiplier is 6, product = ?

12. Multiplicand is 287 tops, multiplier is 9, product = ?

13. If one bale of cotton weighs 456 pounds, what will 6 bales weigh? *Ans.* 2736 pounds.

14. There are 640 acres in a square mile; how many acres in 9 sq. miles? *Ans.* 5760 acres.

15. There are 8 rows in an orchard and 123 trees in each row; how many trees in the orchard? *Ans.* ?

16. A man travels 7 miles; how many yards does he travel, there being 1760 yards in a mile?

Ans. 12320 yards.

17. Which will cost the more; 137 yards at 6 cents a yard, or 117 pounds at 7 cents a pound? *Ans.* ?

111. CASE II.—When the multiplier is any number.

1. Multiply 523 by 746.

EXPLANATION.—Write one of the factors (generally the less) under the other, as in the margin, and, beginning at the right, we say:

3 ones $\times 746 = 2238$ *ones* = 2238,
2 tens $\times 746 = 1492$ *tens* = 14920,
5 hunds. $\times 746 = 3730$ *hunds.* 373000,

which added together make 390158.

OPERATION.

746
523
2238
14920
373000
390158

In practice, the work is done thus: Multiply 746 by 3, 2, 5, in succession as in CASE I, writing the products and their sum as in the margin.

OPERATION.

746
523
2238
1492
3730
390158

The products resulting from multiplying the multiplicand by the *ones*, *tens*, *hundreds*, *etc.*, of the multiplier are called **partial products.**

From the preceding examples and explanations we derive the

RULE.—I. *Write the multiplier under the multiplicand.*

II. *Begin at the right, multiply the multiplicand by the ones, tens, hundreds, etc., of the multiplier, placing the right hand figure of each partial product under the figure of the multiplier used to obtain it, and add the partial products.*

PROOF.—*Multiply the multiplier by the multiplicand, and if the product is the same as before, the work is probably correct.*

In this manner multiply and prove:

(2)	(3)	(4)	(5)
96	85	125	542
15	25	81	27
Ans. 1440	*Ans.* 2125	*Ans.* 10125	*Ans.* 14634

(6)	(7)	(8)
785	1283	845
72	144	108
Ans. 56520	*Ans.* 184752	*Ans.* 91260

Multiply:

9. 155 bushels by 102. *Ans.* 15810 bushels.
10. 275 days by 203. *Ans.* ?
11. 1652 hours by 205. *Ans.* 338660 hours.
12. $15 \times 14 = ?$ *Ans.* 210.
13. $25 \times 18 = ?$ *Ans.* ?
14. $125 \times 23 = ?$ *Ans.* 2875.
15. $168 \times 41 = ?$ *Ans.* 6888.

16. Multiply 4350 by 2700.

OPERATION.

435
27
3045
870
11745000 *Ans.*

112. RULE.—*Omit the 0's on the right of the multiplicand and multiplier, multiply the remaining figures together, and annex the 0's omitted to the result.*

Multiply:

17. 125 by 20. *Ans.* 2500.
18. 348 by 30. *Ans.* 10440.
19. 760 by 40. *Ans.* ?
20. 950 by 60. *Ans.* ?
21. 708000 by 6500. *Ans.* 4602000000.
22. 45600 by 3400. *Ans.* ?

What is the number whose **c** factors are:

23. 648 and 100? *Ans.* ?
24. 21200 and 70? *Ans.* ?
25. 487100 and 27000? *Ans.* 13151700000.
26. 359260 and 3040? *Ans.* 1092150400.
27. A hogshead holds 63 gallons; how many gallons do 49 hogsheads hold? *Ans.* 3087 gallons.
28. If a vessel sails 169 miles a day, how many miles will she sail in 576 days? *Ans.* 97344 miles.
29. If a regiment of soldiers contains 1128 men, how many men are there in an army of 106 regiments? *Ans.* 119568 men.
30. What is the weight of 45 bales of cotton, each bale weighing 463 pounds? *Ans.* 20835 pounds.
31. What is the weight of 74 hogsheads of sugar, each hogshead weighing 1395 pounds? *Ans.* 103230 pounds.
32. If a carriage wheel revolves 419 times in going a mile, how many times will it revolve in going 2 miles? 5 miles? 100 miles? 340 miles? *Ans.* ?
33. What will 27 horses cost at $140 apiece? *Ans.* $3780.
34. If 1 buggy cost $143, what will be the cost of 3 buggies? 10 buggies? 50 buggies? 140 buggies?
35. There are 365 days in a year; how many days are there in 32 years? *Ans.* 11680 days.
36. An orchard contains 240 peach trees. If there are on an average 135 peaches on each tree, how many peaches are there in the orchard? *Ans.* 32400.

37. Suppose a book to contain 470 pages, 45 lines on each page, and 50 letters in each line, how many letters in the book? *Ans.* 1057500.

113. There are:

60 pounds in 1 bushel of wheat.	196 pounds in 1 barrel of flour.
56 pounds in 1 bushel of corn.	200 lbs. in 1 bbl. of pork or beef.
32 pounds in 1 bushel of oats.	280 pounds in 1 barrel of salt.
100 pounds in 1 keg of nails.	180 pounds in 1 barrel of coal.

How many pounds in:

38. 384 bushels of wheat? *Ans.* 23040 pounds.
39. 250 bushels of oats? *Ans.* 8000 pounds.
40. 213 kegs of nails? *Ans.* ?
41. 148 barrels of beef? *Ans.* 29600 pounds.
42. 97 barrels of salt? *Ans.* ?
43. 56 barrels of flour? *Ans.* ?
44. 534 bushels of corn? *Ans.* ?
45. 862 barrels of coal? *Ans.* 155160 pounds.

114. Make problems of the following, taking the terms in order, and giving the answer to each:

46. 420 **h** $\times$ 16 + 180 **h** = ?
47. 518 **c** $\times$ 61 — 2963 **c** = ?
48. 6080 **d** $\times$ 360 — 12384 **d** = ?

115. Parallel Problems.

1.[m] What is the number whose **c** factors are 12 and 7?
2. What is the product of 452 and 25? *Ans.* 11300.
3.[m] What will 9 oranges cost at 8 cents apiece?
4. What will 136 tables cost at $18 per table? *Ans.* $2448.
5.[m] How long will it take 1 man to do a work which 8 men can do in 12 days?

6. If 54 men can build a wall in 36 days, how long will it take 1 man to build it? *Ans.* 1944 days.

7.[m] How many units in 7? If each of them is 5 cts., what will all of them amount to?

8. If each unit of 365 is 24 hours, how many hours are there in all? *Ans.* 8760 hours.

9.[m] What is the number whose **c** parts are \$19 and 12 times \$5?

10. If a boy has \$93 when he is 18 years old, and saves \$125 each year until he is 30 years old, how much money will he then have? *Ans.* \$1593.

11.[m] If a boy gathers 16 nuts each day and eats 7 of them each night, how many nuts will he have in 6 days?

12. A government surveyor receives \$150 a month, and expends \$72; how much does he save in 9 months? *Ans.* \$702.

13.[m] A father has 5 sons; to 2 of them he gave \$8 apiece, and to each of the others \$9; how much did he give them all?

14. 23 boys went out to gather chestnuts. They separated into two squads, 15 boys going in the first squad. Each boy of the first squad gathered 324 chestnuts, and each of the second 245 chestnuts; how many chestnuts were gathered in all? *Ans.* 6820 chestnuts.

15.[m] A boy had 5 boxes, and in each box 5 oranges, and sold each orange for 5 cents; how much did he receive for all?

16. A load of 12 bales of cotton, each bale weighing 450 pounds, was sold at 9c. a pound; what was the sum received for the load? *Ans.* 48600 cents.

17.[m] What is the number whose **c** parts are 9 times \$5 and \$15?

18. A drover bought 48 sheep at \$2.50 a head, and

sold them for $23.25 more than the cost; what did he receive for them? *Ans.* $143.25.

19.m What is the number whose three **c** parts are 2 times 3, 4 times 5, and 6 times 10?

20. A merchant bought 24 sets of crockery of 45 pieces each, 29 sets of 37 pieces each, and 54 sets of 60 pieces each; how many pieces did he buy? *Ans.* 5393.

21.m If John eats 7 biscuits in one day, how many will he eat in a month of 30 days?

22. A common clock strikes 156 times every day; how many times does it strike in a year of 365 days? *Ans.* 56940.

23.m How much more is 12 times 5 than 28 and 17 added together?

24. What is the *difference* between the *product* of 375 and 120, and the *sum* of 28507 and 13629? *Ans.* 2864.

25.m If the *multiplicand* is the *sum* of 8 and 7, and the *multiplier* the *difference* between 13 and 4, what is is the *product?*

26. $(381 + 264) \times (583 - 196) = ?$ *Ans.* 249615.

116. Questions for Review.

What is: 1. Multiplication? 2. The multiplicand? 3. The multiplier? 4. The product? 5. The Sign of Multiplication?

What is denoted by the sign $\times$?

State the three principles of Multiplication.

What are the **c** factors of a number? What do we find by Multiplication?

How is a number multiplied by 10, 100, etc.?

What is the rule for Multiplication when the multiplier is: 1. A digit? 2. Any whole number?

DIVISION.

INDUCTIVE EXERCISES.

117. Are 3 and 3 complemental parts of 6? Does 6 contain its parts? How many 3's does 6 contain? 3 is contained in 6 how many times? How many times can 3 be subtracted from 6?

How many times is 4 contained in 20?

Ans. As many times as 4 can be subtracted from 20.

$20-4=16$, 1 time; $16-4=12$, 2 times; $12-4=8$, 3 times; $8-4=4$, 4 times; $4-4=0$, 5 times.

How many times is 3 contained in 18? 5 contained in 30? 7 contained in 28? 9 contained in 45? 11 contained in 66? 12 contained in 84?

A man divided \$8 equally among his sons, giving each \$2; how many sons had he? \$2 is contained in \$8 how many times?

A horse traveled 28 miles at the rate of 4 miles per hour; how many hours did he travel? 28 contains 4 how many times?

James has 20 marbles and wishes to put them in boxes so as to have 5 marbles in each box; how many boxes does he need? 20 contains 5 how many times?

How many times is 5 contained in 17?

$17-5=12$, 1 time; $12-5=7$, 2 times; $7-5=2$, 3 times.

Ans. 3 *times* and 2 *over*.

How many times is 4 contained in 23? 6 in 31? 7 in 46? 8 in 27? 9 in 33?

Definitions

118. Division is the process of finding how many times one number is contained in another.

119. The number to be divided is the **Dividend,** the number by which it is divided the **Divisor,** and the result the **Quotient.**

120. When a part of the dividend is left after the division is performed, it is the **Remainder,** and must always be *less* than the *divisor.*

121. The Sign of Division is $\div$. It is read: *divided by,* and shows that the number *before* it is to be divided by the number *after* it.

Thus, $35 \div 7 = 5$ is read: 35 divided by 7 equals 5. This is also read: 35 contains 7 5 times. Here, 35 is the *dividend,* 7 the *divisor,* and 5 the *quotient.* $35 \div 7$ is sometimes written 7)35, and $\frac{35}{7}$.

122. Copy and read:

1. $12 \div 3 = 4$.
2. $9)18 = 2$.
3. $\frac{1}{2}2 = 6$.
4. 24 men $\div$ 6 men $= 4$.
5. 56 boys $\div 7 = 8$ boys.
6. 3)27 weeks $= 9$ weeks.

Point out the *dividend, divisor,* and *quotient* in each.

123. Express by signs:

1. 35 contains 7, 5 times.
2. 72 contains 8, 9 times.
3. \$28 contains \$4, 7 times.
4. 12 divided by 4 is 3.
5. 45 contains 9 how many times?
6. 24 pens divided by 6 equals 4 pens.

124. Principles.

1°. When the divisor and dividend are *like* numbers, the quotient is an *abstract* number.

2°. When the divisor is an *abstract* number, the quotient and dividend are *like* numbers.

3°. When the divisor and dividend are like numbers, division may be effected by subtraction.

Let the pupil point out, or verify, these principles in each example of Art. **122.**

Relation of Division to Multiplication.

125. 4 times 3 are how many? 3 is contained in 12 how many times? 4 is contained in 12 how many times?

4 times *what* are 12? *Ans.* 3, because $12 \div 4 = 3$.

What times 3 are 12? *Ans.* 4, because $12 \div 3 = 4$.

Questions and Answers.

$5 \times ? = 20$? *Ans.* 4, because $20 \div 5 = 4$.

$? \times 6 = 42$? *Ans.* 7, because $42 \div 6 = 7$.

In a similar manner answer the following, performing the division, if necessary, by subtraction:

$7 \times ? = 21$. $? \times 6 = 42$. 9 men $\times ? = 36$ men.

$5 \times ? = 30$. $8 \times ? = 56$. $? \times 4$ cents $= 32$ cents.

Principles.

1°. Division is the *reverse* of multiplication.

2°. By division we find one of the complemental factors of a number, when the number and the other factor are given.

126. Since Division is the *reverse* of multiplication, by reversing the table of the latter, we have the following table, which may be read thus: 1 in none, no times; 1 in 1, one time; 1 in 2, two times, etc.

Division Table.

1			2			3			4		
1 in	0 =	0	2 in	0 =	0	3 in	0 =	0	4 in	0 =	0
1 in	1 =	1	2 in	2 =	1	3 in	3 =	1	4 in	4 =	1
1 in	2 =	2	2 in	4 =	2	3 in	6 =	2	4 in	8 =	2
1 in	3 =	3	2 in	6 =	3	3 in	9 =	3	4 in	12 =	3
1 in	4 =	4	2 in	8 =	4	3 in	12 =	4	4 in	16 =	4
1 in	5 =	5	2 in	10 =	5	3 in	15 =	5	4 in	20 =	5
1 in	6 =	6	2 in	12 =	6	3 in	18 =	6	4 in	24 =	6
1 in	7 =	7	2 in	14 =	7	3 in	21 =	7	4 in	28 =	7
1 in	8 =	8	2 in	16 =	8	3 in	24 =	8	4 in	32 =	8
1 in	9 =	9	2 in	18 =	9	3 in	27 =	9	4 in	36 =	9
1 in	10 =	10	2 in	20 =	10	3 in	30 =	10	4 in	40 =	10
1 in	11 =	11	2 in	22 =	11	3 in	33 =	11	4 in	44 =	11
1 in	12 =	12	2 in	24 =	12	3 in	36 =	12	4 in	48 =	12

5			6			7			8		
5 in	0 =	0	6 in	0 =	0	7 in	0 =	0	8 in	0 =	0
5 in	5 =	1	6 in	6 =	1	7 in	7 =	1	8 in	8 =	1
5 in	10 =	2	6 in	12 =	2	7 in	14 =	2	8 in	16 =	2
5 in	15 =	3	6 in	18 =	3	7 in	21 =	3	8 in	24 =	3
5 in	20 =	4	6 in	24 =	4	7 in	28 =	4	8 in	32 =	4
5 in	25 =	5	6 in	30 =	5	7 in	35 =	5	8 in	40 =	5
5 in	30 =	6	6 in	36 =	6	7 in	42 =	6	8 in	48 =	6
5 in	35 =	7	6 in	42 =	7	7 in	49 =	7	8 in	56 =	7
5 in	40 =	8	6 in	48 =	8	7 in	56 =	8	8 in	64 =	8
5 in	45 =	9	6 in	54 =	9	7 in	63 =	9	8 in	72 =	9
5 in	50 =	10	6 in	60 =	10	7 in	70 =	10	8 in	80 =	10
5 in	55 =	11	6 in	66 =	11	7 in	77 =	11	8 in	88 =	11
5 in	60 =	12	6 in	72 =	12	7 in	84 =	12	8 in	96 =	12

9			10			11			12		
9 in	0 =	0	10 in	0 =	0	11 in	0 =	0	12 in	0 =	0
9 in	9 =	1	10 in	10 =	1	11 in	11 =	1	12 in	12 =	1
9 in	18 =	2	10 in	20 =	2	11 in	22 =	2	12 in	24 =	2
9 in	27 =	3	10 in	30 =	3	11 in	33 =	3	12 in	36 =	3
9 in	36 =	4	10 in	40 =	4	11 in	44 =	4	12 in	48 =	4
9 in	45 =	5	10 in	50 =	5	11 in	55 =	5	12 in	60 =	5
9 in	54 =	6	10 in	60 =	6	11 in	66 =	6	12 in	72 =	6
9 in	63 =	7	10 in	70 =	7	11 in	77 =	7	12 in	84 =	7
9 in	72 =	8	10 in	80 =	8	11 in	88 =	8	12 in	96 =	8
9 in	81 =	9	10 in	90 =	9	11 in	99 =	9	12 in	108 =	9
9 in	90 =	10	10 in	100 =	10	11 in	110 =	10	12 in	120 =	10
9 in	99 =	11	10 in	110 =	11	11 in	121 =	11	12 in	132 =	11
9 in	108 =	12	10 in	120 =	12	11 in	132 =	12	12 in	144 =	12

Drill Exercises.

127. These exercises should be studied by the pupils. They should also be written on the board, and used in class drill daily, until every pupil can call all the quotients instantly.

2)4	10	18	6	12	8	14	20	16	2
3)3	27	9	21	15	12	18	6	24	30
4)12	28	20	16	24	8	32	40	4	36
5)45	15	35	25	20	30	10	40	50	5
6)60	48	12	36	24	30	42	18	54	6
7)28	42	14	56	70	35	7	63	21	49
8)24	56	40	32	48	16	64	80	8	72
9)90	72	18	54	36	45	63	27	63	9

Mental Exercises.

128. These exercises should be performed by Subtraction and Division until the relation of the two operations is clearly apprehended.

Case I.—When there is no remainder.

1. How many 2's in 4? 3's in 3? 4's in 12? 5's in 45? 6's in 60? 7's in 28? 8's in 24? 9's in 90?

2. Divide 10 by 2; 27 by 3; 28 by 4; 15 by 5; 48 by 6; 42 by 7; 56 by 8; 72 by 9.

3. 2)2 = ? 2)18 = ? 3)9 = ? 4)20 = ? 5)35 = ?

4. $\frac{6}{2} = ?$ $\frac{21}{3} = ?$ $\frac{16}{4} = ?$ $\frac{25}{5} = ?$ $\frac{36}{6} = ?$ $\frac{56}{7} = ?$

5. $2 \times ? = 12$. $3 \times ? = 15$. $4 \times ? = 36$. $5 \times ? = 20$. $6 \times ? = 24$. $? \times 7 = 70$. $? \times 8 = 48$. $? \times 9 = 36$. $? \times 5 = 15$. $? \times 7 = 49$.

6. How many times 7 feet make 42 feet?

7. How many times 3 days make 15 days?

8. How many times are 6 cents contained in 54 cents?

9. How many times are 8 bushels contained in 56 bushels?

10. I bought 30 cents worth of oranges and paid 5 cents for each orange; how many did I buy?

11. If 6 yards of cloth make 1 suit, how many suits can be made of 60 yards?

12. How many hours will it take a man to walk 21 miles, if he walks 3 miles per hour?

13. There are 40 boys in school, and 8 boys in each class; how many classes are there?

14. The dividend is 54, the divisor 6; what is the quotient?

15. The dividend is 63, the divisor 9; what is the quotient?

16. 8 is a factor of 40; what is the **c** factor?

17. 7 is a factor of 42; what is the **c** factor?

129. Case II.—When there is a remainder.

18. How many times is 8 contained in 51?

Operation.—The next number below 51 that 8 will divide evenly is 48, which contains 8 6 times; and since 51 is 3 more than 48, we say: 8 is contained in 51 6 times and 3 over.

How many times is:

19. 2 contained in 5? 3 in 5 ? 4 in 15? 5 in 49?

20. 6 contained in 65? 7 in 34? 8 in 31? 9 in 98?

21. 2 contained in 11? 3 in 28? 4 in 30? 5 in 17?

22. 6 contained in 50? 7 in 45? 8 in 60? 9 in 80?

23. 2 contained in 9? 3 in 14? 4 in 11? 5 in 32?

24. 6 contained in 35? 7 in 39? 8 in 21? 9 in 47?

25. 2 contained in 21? 3 in 7? 4 in 43? 5 in 43?

26. 6 contained in 23? 7 in 67? 8 in 86? 9 in 33?

Exercises in Making Problems.

130. 1. Make a problem of: 10 **d** ÷ 5 = ?

Ans. 10 dollars divided by 5 equals how many? *Or*, If 5 apples cost 10 dimes, how much will 1 apple cost?

Make a problem of:

2. 20 **c** ÷ 4 = ?
3. 42 **m** ÷ 6 = ?
4. 63 **a** ÷ 7 = ?
5. 54 **b** ÷ 9 = ?

6. Make a problem of: 10 **c** × 4 ÷ 8 **c** = ?

Ans. 10 cups taken 4 times contain 8 cups how many times?

Make a problem of:

7. 9 **b** × 4 ÷ 6 **b** = ? *Ans.* ?
8. 8 **g** × 6 ÷ 12 **g** = ? *Ans.* ?
9. Make a problem of: 7 **b** × 4 + 2 **b** ÷ 5 **b** = ?

Ans. 7 books taken 4 times and 2 books more contain 5 books how many times?

Make problems of the following, taking the terms in order:

10. 9 **c** × 2 + 2 **c** ÷ 5 **c** = ? *Ans.* 4.
11. 4 **m** × 5 − 6 **m** ÷ 7 **m** = ? *Ans.* 2.
12. 9 **h** × 7 + 3 **h** ÷ 6 **h** = ? *Ans.* ?
13. 10 **d** × 4 − 5 **d** ÷ 7 **d** = ? *Ans.* ?

Equal Parts.

131. When a number is divided:

Into *two* equal parts, one of the parts is 1 *half* of the number;

Into *three* equal parts, one of the parts is 1 *third* of the number;

Into *four* equal parts, one of the parts is 1 *fourth* of the number;

Into *five* equal parts, one of the parts is 1 *fifth* of the number, and so on.

The parts 1 half, 1 third, 1 fourth, 1 fifth, etc., are written: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.

To find 1 *half* of a number: Divide the number by 2.

To find 1 *third* of a number: Divide the number by 3.

To find 1 *fourth* of a number: Divide the number by 4.

To find 1 *fifth* of a number: Divide the number by 5.

Mental Exercises.

132. 1. How much is: 1 *half* of 16? 1 *third* of 24? 1 *fourth* of 36? 1 *fifth* of 50? 1 *sixth* of 54? 1 *seventh* of 21? 1 *eighth* of 72? 1 *ninth* of 27? 1 *half* of 14? 1 *third* of 18? 1 *fourth* of 32? 1 *fifth* of 30? 1 *sixth* of 42? 1 *seventh* of 70?

2. How much is: $\frac{1}{2}$ of 12? $\frac{1}{3}$ of 15? $\frac{1}{4}$ of 24? $\frac{1}{5}$ of 20? $\frac{1}{6}$ of 36? $\frac{1}{7}$ of 49? $\frac{1}{8}$ of 48? $\frac{1}{9}$ of 54?

3. 6 feet are in 2 yards; how many feet are in 1 yard? *Ans.* $\frac{1}{2}$ of 6 feet, or 3 feet.

4. 20 inches in 4 hands; how many inches in 1 hand?

5. 35 days in 5 weeks; how many days in 1 week?

6. 30 yards in 6 suits; how many yards in 1 suit?

7. 42 chairs in 7 sets; how many chairs in 1 set?

8. If 8 hats cost \$24, what is the cost of 1 hat?

9. If 7 guns cost \$63, what is the cost of 1 gun?

10. If 6 peaches cost 12 cents, what is the cost of 1 peach?

11. If 9 pounds of sugar cost 90 cents, what is the cost of 1 pound?

12. If 10 gallons of whisky cost \$30, what is the cost of 1 gallon?

13. If a horse walks 42 miles in 7 hours, how many miles does he travel in 1 hour?

14. If 8 men earn $32 in a day, how many dollars does 1 man earn?

15. If 12 pairs of shoes cost $36, what will 1 pair of shoes cost?

16. Five boys gathered 40 pints of chestnuts, which they shared equally; how many pints had each boy?

17. How far must a man travel per hour to go 24 miles in 6 hours?

18. If 10 acres produce 100 bushels, how many bushels is that per acre?

19. How much is one-sixth of 30 feet?

20. One-fifth of a pole 35 feet long was broken off; how many feet were broken off?

21. Is 5 one-fifth or one-fourth of 20?

22. If 8 yards of cloth cost 56 cents, what is the cost of 1 yard?

23. If 9 hats cost $36, what is the cost of 1 hat?

24. If 12 horses consume 72 bushels of corn, how much will 1 horse consume in the same time?

25. A fox is 54 feet ahead of a hound; if the hound gains on him 6 feet in every minute, in how many minutes will he overtake the fox?

26. Two boats on the Mississippi R. are 50 miles apart. The hindmost boat gains on the other 5 miles an hour; in how many hours will it overtake the other?

27. If 1 pipe discharges a cistern of water in 84 hours, how long will it take 7 pipes of the same size to discharge the cistern?

28. 4 times 9 are how many times 6? 12? 3?

29. 6 times 8 are how many times 12? 4?

30. How many times is 10 contained in 5 times 6?

31. How many times is 6 contained in 5 times 12?

32. How many times is 4 contained in 19 + 5?

33. How many times is 7 contained in 40 — 5?

34. How many times is 3 contained in 21? \$4 contained in \$28? 5 bushels contained in 45 bushels? 6 tens contained in 42 tens? 7 **a** contained in 56 **a**? 9 **u** contained in 108 **u**? 8 fifths contained in 64 fifths?

35. How much is 1 *fourth* of 16? Of \$20? Of 20 **u**?

36. How much is $\frac{1}{5}$ of 15c.? $\frac{1}{6}$ of 18 cows? $\frac{1}{7}$ of 28 **b**? $\frac{1}{3}$ of 9 *men?* $\frac{1}{3}$ of 9 *tens?* $\frac{1}{3}$ of 9 **u**? $\frac{1}{3}$ of 9 *sevenths?*

Written Exercises.

133. Case I.—When each figure of the dividend is divisible by the divisor.

1. Divide 369 by 3.

Explanation.—369 = 3 *hunds.*, 6 *tens*, 9 *ones*, which divided by 3 gives 1 *hund.*, 2 *tens*, 3 *ones*, or 123.

OPERATION.

$$3)369$$
$$123$$

Hence, the

Rule.—*Place the divisor on the left of the dividend, divide each figure of the latter by the former, and write the quotient under the figure divided.*

Divide:

2. 264 by 2. *Ans.* 132.
3. 84 by 4. *Ans.* 21.
4. 906 by 3. *Ans.* 302.
5. 3063 by 3. *Ans.* 1021.
6. 4804 by 4. *Ans.* ?
7. 2846 by 2. *Ans.* ?

8. What will 1 book cost if 3 books cost 63 cents?

Ans. 21 cents.

9. What will one lot cost if 2 lots cost \$248?

Ans. \$124.

10. How far does a railroad train go in 1 hour if it travels 96 miles in 3 hours? *Ans.* 32 miles.

11. A father divided 8084 chestnuts equally among 4 boys; how many did each boy get? *Ans.* ?

134. CASE II.—When each figure of the dividend can not be divided evenly by the divisor.

1. Divide 11352 by 3.

Now, we may arrange the dividend so that the tens, hunds., etc., shall each be divisible by the divisor, and then divide as in CASE I.

OPERATION.

$$\begin{aligned} 11352 &= 11 \text{ thous. } 3 \text{ hunds. } 5 \text{ tens, } 2 \text{ ones.} \\ &= 9 \text{ thous. } 23 \text{ hunds. } 5 \text{ tens, } 2 \text{ ones.} \\ &= 9 \text{ thous. } 21 \text{ hunds. } 25 \text{ tens, } 2 \text{ ones.} \\ &= 9 \text{ thous. } 21 \text{ hunds. } 24 \text{ tens, } 12 \text{ ones.} \end{aligned}$$

$$\begin{array}{r} 3)\overline{9 \text{ thous. } 21 \text{ hunds. } 24 \text{ tens, } 12 \text{ ones.}} \\ 3 \text{ thous. } 7 \text{ hunds. } 8 \text{ tens, } 4 \text{ ones} = 3784. \end{array}$$

EXPLANATION.—The first number below 11 thous. that is divisible by 3 is 9 thous., which we write in the place of 11 thous. and prefix the 2 over to 3, making 23 hunds.

Again, the first number below 23 hunds. that is divisible by 3 is 21 hunds., which we write in the place of 23 hunds., and prefix the 2 over to 5, making 25 tens. Again, the first number below 25 tens that is divisible by 3 is 24 tens, which we write in the place of 25 tens, and prefix the 1 over over to 2, making 12 ones. Now dividing by 3, as in CASE I, we have 3 thous. 7 hunds. 8 tens. 4 ones, or 3784.

SUGGESTIONS TO THE TEACHER.—This method of performing Division is a splendid exercise for beginners. It not only prepares them for the mechanical operations of Division, but leads them into a clear conception of the principles on which the operations depend. The parenthesis may be used to separate the tens, hundreds, etc., if preferred, as in the following example:

2. Divide 13572 by 4.

OPERATION.

1st arrangement,	(13)	(5)	(7)	(2)	
2d "	(12)	(15)	(7)	(2)	
3d "	(12)	(12)	(37)	(2)	
4th "	(12)	(12)	(36)	(12)	
Dividing through by 4,	3	3	9	3,	or 3393, *Ans.*

Divide:

3. 108 by 3.	*Ans.* 36.	7. 1401 by 3.	*Ans.* 467.
4. 172 by 4.	*Ans.* 43.	8. 4325 by 5.	*Ans.* 865.
5. 280 by 5.	*Ans.* 56.	9. 7230 by 10.	*Ans.* 723.
6. 342 by 6.	*Ans.* 57.	10. 34120 by 8.	*Ans.* ?

Short Division.

135. When the divisor is 12 or less, we use short division. That is, we perform the operations mentally, and write the results only.

1. Divide 387586 by 9.

OPERATION.

9)387586

43065 Rem. 1.

Explanation. — Write the divisor and dividend as in the margin. We now say 9 in 38, 4 times and 2 over; write 4 below and prefix 2 to 7, making 27. Again, 9 in 27, 3 times and 0 over; write 3 below, and nothing to carry to the 5. Hence, 9 in 5, 0 times and 5 over; write 0 below and prefix 5 to 8, making 58. Again, 9 in 58, 6 times and 4 over; write 6 below and prefix 4 to 6, making 46. Again, 9 in 46, 5 times and 1 over; write 5 below and 1 Rem. to the right. The pupil should be trained to call results only. Thus, 4, 3, 0, 6, 5, and Rem. 1.

From the preceding articles we derive the

Rule—I. *Write the divisor at the left of the dividend with a line between.*

II. *Find how many times the divisor is contained in the first left-hand figure or figures of the dividend, and write the quotient underneath, and so proceed with each figure.*

III. *If there is a remainder, prefix it to the next figure of the dividend, and divide as before.*

IV. *When the divisor is not contained in any partial dividend, write a cipher in the quotient, and prefix this number to the next figure of the dividend, and divide as before.*

PROOF.—*Multiply the quotient by the divisor, and to the product add the remainder, if any; the result should be equal to the dividend.*

EXERCISES.

Divide:

2. 3754 by 2.	*Ans.* 1877.	9. 95167 by 3.	*Rem.* 1.
3. 2871 by 3.	*Ans.* 957.	10. 12678 by 4.	*Rem.* 2.
4. 7508 by 4.	*Ans.* 1877.	11. 75613 by 5.	*Rem.* 3.
5. 6730 by 5.	*Ans.* 1346.	12. 14789 by 6.	*Rem.* 5.
6. 6102 by 6.	*Ans.* 1017.	13. 95328 by 7.	*Rem.* 2.
7. 3402 by 7.	*Ans.* 486.	14. 18903 by 9.	*Rem.* 3.
8. 10024 by 8.	*Ans.* 1253.	15. 74638 by 10.	*Rem.* 8.
16. $2643 \div 3 = ?$	*Ans.* 881.	18. $\frac{4572}{7} = ?$	*Rem.* 1.
17. $6235 \div 5 = ?$	*Ans.* ?	19. $\frac{5875}{8} = ?$	*Rem.* ?

20. How many yards of cloth will it take, at 5 cents a yard, to amount to 37295 cents? *Ans.* 7459 yards.

21. A merchant spent \$33224 for hats, paying, on an average \$4 apiece; how many hats did he buy?

Ans. 8306 hats.

22. A farmer received \$1950 for a lot of land which he sold at the rate of \$6 per acre; how many acres did he sell? *Ans.* ?

23. 4308 chestnuts were divided equally among 6 boys; how many chestnuts did each boy receive? *Ans.* ?

24. There are 12320 yards in 7 miles; how many yards in one mile? *Ans.* 1760.

25. I counted the legs of all the horses in a drove, and found that there were 476; how many horses were in the drove? *Ans.* ?

Long Division.

136. When the divisor is greater than 12 we write down all the figures employed, and call the operation Long Division.

137. Case I.—When the quotient is not greater than 9.

1. Divide 91 by 21.

Explanation. — The first figure of the dividend is 9, and that of the divisor is 2; 2 in 9, 4 times. Place the 4 on the right, multiply it by 21, subtract the product 84 from 91, which gives 7 remainder.

operation.

$$\begin{array}{r} 21)91(4 \\ \underline{84} \\ 7 \end{array}$$

2. Divide 442 by 75.

The first part of the dividend that contains the first figure (7) of the divisor is 44; 7 in 44, 6 times. Place the 6 on the right, multiply it by 75, and since the result, 450, is *larger* than 442, 6 is *too large*. Hence, we take the next less number (5), put it in place of 6, multiply it by 75, and since the result, 375, is *less* than 442, 5 is the correct quotient, and the answer is: quo. 5 rem. 67.

operation.

$$\begin{array}{r} 75)442(6 \\ \underline{450} \end{array}$$

$$\begin{array}{r} 75)442(5 \\ \underline{375} \\ 67 \end{array}$$

3. Divide 337 by 35.

Since 33 is less than 35, we say 3 in 33, 11 times. But in dividing in this manner we can never get a greater quotient than 9; hence, instead of 11 we write 9 on the right, multiply it by 35, and, since the result, 315, is less than 337, 9 is the correct quotient.

$$\begin{array}{r} 35)337(9 \\ \underline{315} \\ 22 \end{array}$$

From the preceding work we derive the

RULE.—I. *Divide the first figure or figures of the dividend by the first figure of the divisor; place the result on the right and call it the trial quotient.*

II. *Multiply the divisor by the trial quotient and place the product under the dividend; if it is larger than the dividend, the trial quotient is too large and must be diminished; if it is smaller, subtract it from the dividend, and if the remainder is less than the divisor the work is correct; if greater, the trial quotient is too small and must be increased.*

PROOF.—*The same as in short division.*

Divide:

4. 117 by 23.	*Ans. Q.* 5, *R.* 2.	10. 300 by 45.	*Rem.* 30.
5. 400 by 76.	*Rem.* 20.	11. 967 by 98.	*Rem.* 85.
6. 311 by 88.	*Rem.* 47.	12. 573 by 75.	*Rem.* 48.
7. 728 by 93.	*Rem.* 77.	13. 805 by 237.	*Rem.* 94.
8. 643 by 75.	*Quo.* 8.	14. 933 by 465.	*Rem.* 3.
9. 340 by 49.	*Quo.* 6.	15. 1080 by 135.	*Rem.* 0.

16. Divide 43657 by 8705. *Rem.* 132.

17. Divide \$34637 by \$9604. *Rem.* \$5825.

18. Divide 517694 **a** by 89325 **a**. *Rem.* 71069 **a**.

138. CASE II.—When the quotient is more than 9.

1. Divide 9156 by 21.

OPERATION.

21)9156(436
84
75
63
126
126

EXPLANATION.—21 in 91, by CASE I, goes 4 times and rem. 7. Place 4 on the right and annex 5 to 7, making 75. By CASE I, 21 in 75, 3 times and rem. 12. Place 3 on the right and annex 6 to 12, and we find by CASE I, 21 in 126 goes 6 times, which place on the right.

2. Divide 876000 by 125.

OPERATION.

125)876000(7008
875
1000
1000

By CASE I, 125 in 876, 7 times and rem. 1. Place 7 on the right and annex 0, the next figure of the dividend to 1, making 10. Now 125 in 10, 0 times and rem. 10. Place 0 on the right and annex the next figure 0 to 10, making 100. Now 125 in 100, 0 times and 100 rem. Place 0 on the right and annex the next figure 0 to 100, making 1000. By CASE I, 125 in 1000, 8 times, which we place on the right.

From the foregoing examples and operations we derive the following

RULE.—I. *Write the divisor on the left of the dividend, with a line between them, and draw a line on the right.*

II. *Find how many times the divisor is contained in the least number of the left hand figures of the dividend that will contain it, and place the quotient on the right.*

III. *Multiply the divisor by this quotient figure, subtract the product from the figures of the dividend used, and to the remainder annex the next figure of the dividend.*

IV. *Divide as before, and continue the operation until all the figures of the dividend have been brought down.*

V. *When one of the partial dividends is less than the divisor, write 0 for the next figure of the quotient, and bring down the next figure of the dividend.*

PROOF.—*Add the remainder to the product of the divisor and quotient; the result should be equal to the dividend.*

NOTE.—When there is a remainder after all the figures of the dividend have been brought down and divided, it may either be set off by itself, or it may be written over the divisor and annexed to the quotient.

Divide:

3. 625 by 25. *Ans.* 25.
4. 759 by 33. *Ans.* 23.
5. 864 by 36. *Ans.* 24.
6. 882 by 42. *Ans.* 21.
7. 270 by 18. *Ans.* 15.
8. 1778 by 14. *Ans.* 127.
9. 2169 by 18. *Rem.* 9.
10. 3639 by 27. *Rem.* 21.
11. 7540 by 59. *Rem.* 47.
12. 35645 by 215. *Rem.* 170.
13. 58650 by 425. *Quo.* 138. *Rem.* ?
14. 98629 by 687. *Ans.* $143\frac{388}{687}$.
15. 75863 by 3421. *Ans.* $22\frac{601}{3421}$.
16. 10000 by 2749. *Ans.* $3\frac{1753}{2749}$.
17. \$132 by \$27. *Ans.* $4\frac{24}{27}$.
18. \$457 by 56. *Ans.* $\$8\frac{9}{56}$.

19. How many hours will it take a railway-train to go 800 miles at the rate of 32 miles an hour?

Ans. 25 hours.

20. What is the weight of a bale of cotton if 25 bales weigh 11400 pounds? *Ans.* 456 pounds.

21. How many hogsheads of sugar will it take to weigh 17400 pounds, if 1 hogshead weighs 1450 pounds?

Ans. 12.

22. The average price of a drove of horses is \$127, and the price of the whole drove is \$13335; how many horses are in the drove? *Ans.* 105 horses.

23. With \$19608, how many cows can I buy at \$43 a head? *Ans.* 456.

24. How many bales, each weighing 475 pounds, can be made of 93100 pounds of cotton? *Ans.* 196 bales.

25. William can haul 1248 pebbles in his wagon; how many trips will he have to make to haul off 91104 pebbles? *Ans.* 73.

26. If the distance around a wheel is 56 inches, how many times will the wheel turn over in going a distance of 7504 inches? *Ans.* 134.

27. At what price per head must I sell 148 sheep to receive \$1036? *Ans.* \$7.

28. The salary of the President of the United States is \$50000 a year; how much is that a day, there being 365 days in 1 year? *Ans.* $\$136\frac{360}{365}$.

29. There are 56 pounds in a bushel of corn; how many bushels in 12345 pounds? *Ans.* $220\frac{25}{56}$.

139. Make problems of the following, taking the terms in order, and giving the answer to each:

30. 26992 **h** ÷ 482 **h** = ?

31. 3212 **m** + 868 **m** ÷ 34 **m** = ?

32. 4120 **d** − 520 **d** ÷ 150 = ?

33. 703 **p** × 8 − 204 **p** ÷ 125 **p** = ?

Contractions in Division.

140. Case I.—When the divisor is 10, 100, 1000, etc.

1. Divide 1625 by 100.

Dividing according to the rule of Long Division, we obtain the quotient 16 and remainder 25.

Now we observe that this answer could have been obtained by simply cutting off the last two figures (25) of the dividend for a remainder, and taking the balance of the dividend for a quotient.

OPERATION.

100)1625(16
100
625
600
25

Hence, to divide by 10, 100, etc., we have the

Rule.—*Cut off from the right of the dividend as many figures as there are ciphers at the right of the divisor; the remaining figures of the dividend will be the quotient, and those cut off on the right will be the remainder.*

2. Divide 375 by 10. *Ans.* 37, *Rem.* 5.

3. Divide 4316 by 100. *Ans.* 43, *Rem.* 16.
4. Divide 60524 by 1000. *Ans.* 60, *Rem.* 524.
5. How much is 1 tenth of 43? 1 hundredth of 471?
6. There are 10 dimes in one dollar; how many dollars in 40 dimes? 260 dimes? 500 dimes?
7. A farmer having $3254, bought horses at $100 each; how many horses did he buy, and how many dollars had he left?
8. A dealer has 1895 cigars, and wishes to put them in boxes of 100 cigars each; how many boxes does he need, and how many cigars will he have left over?

141. CASE II.—When the divisor is any number with ciphers annexed.

1. Divide 73153 by 2700.

Dividing by the rule of Long Division, we obtain the quotient 27 and remainder 253, which result could have been obtained thus:

Cut off the two 0's of the divisor and the last two figures of the dividend; divide the remaining figures of the dividend (731) by the remaining figures of the divisor (27), and to the remainder (2) annex the two figures cut off (53) for the true remainder.

OPERATION.

$$\begin{array}{r} 27,00)731,53(27 \\ 54 \\ \hline 191 \\ 189 \\ \hline 253 \end{array}$$

Hence, we have the

RULE.—*Cut off the ciphers from the divisor, and also cut off the same number of figures from the right of the dividend; divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder, if any, annex the figures cut off from the dividend for a true remainder.*

2. Divide 37657 by 50. *Ans.* 753, *Rem.* 7.
3. Divide 43787 by 600. *Ans.* 72, *Rem.* 587.
4. Divide 35016 by 700. *Ans.* 50, *Rem.* 16.

5. Divide 63242 by 3500. *Ans.* 18, *Rem.* 242.
6. Divide 71831 by 6400. *Ans.* $11\frac{1431}{6400}$.
7. Divide 93045 by 17000. *Ans.* $5\frac{8045}{17000}$.
8. Divide 184973 by 23000. *Ans.* $8\frac{9733}{23000}$.
9. Divide 846 by 40. *Ans.* ?
10. Divide 7593 by 900. *Ans.* ?
11. Divide 23956 by 3700. *Ans.* ?

12. If 40 barrels of molasses cost $480, what is the price of 1 barrel? *Ans.* $12.

13. A farmer sold 600 acres of land for $7800; how much was that per acre? *Ans.* $13.

14. A merchant sold 8000 yards of cloth for 184000 cents; how much was that per yard? *Ans.* 23 cents.

15. John and Henry gather 6275 chincapins, and desire to put them in sacks containing 290 chincapins each; how many sacks do they need, and how many chincapins will be left over?

Mental Exercises.

An important class of problems involving Multiplication and Division.

142. 1. If 4 yards of cloth cost 20 cents, what will 7 yards cost at the same rate?*

Explanation.—Since 4 yards cost 20 cents we divide 20 by 4 to get the cost of 1 yard, which gives 5 cents. Now, since 1 yard cost 5 cents, we multiply 5 by 7 to get the cost of 7 yards.

OPERATION.

$$\begin{array}{r} 4)20 \\ \hline 5 \\ 7 \\ \hline 35 \text{ cents.} \end{array}$$

2. If 5 yards of cloth cost 40 cents, what will 8 yards cost? *Ans.* 64 c.

* The words "at the same rate" are supposed to follow several of the following exercises.

3. If 4 apples cost 12 cents, what will 9 apples cost?

4. If 6 peaches cost 24 cents, what will 10 peaches cost? *Ans.* 40 c.

5. If 7 melons cost 70 cents, what will 9 melons cost?

6. If 9 hats cost \$27, what will 7 hats cost? *Ans.* \$21.

7. If 8 books cost \$16, what will 11 books cost?

8. If 3 *tens* cost \$15, what will 7 *tens* cost? *Ans.* \$35.

9. If 2 *threes* cost \$24, what will 5 *threes* cost?

10. If 5 *fourths* cost \$30, what will 3 *fourths* cost?

11. If 4 **u** cost 24 cents, what will 9 **u** cost? *Ans.* ?

12. If 4 boys kill 8 squirrels, how many will 5 boys kill? *Ans.* ?

13. If 5 cats catch 15 rats, how many will 7 cats catch? *Ans.* ?

14. If 6 boys eat 18 biscuits, how many will 12 boys eat? *Ans.* ?

15. If 5 girls have 40 fingers, how many fingers have 7 girls? *Ans.* ?

16. If 7 horses have 28 feet, how many feet have 12 horses? *Ans.* ?

17. If 5 gallons = 20 quarts, then 8 gallons = ? *Ans.* 32 quarts.

18. If 4 quarts = 8 pints, than 11 quarts = ?

19. If 6 dimes = 60 cents, then 13 dimes = ?

20. If \$9 = 90 dimes, then \$6 = ? *Ans.* 60 dimes.

Written Exercises.

1. If 18 acres cost \$270, what will 20 acres cost? *Ans.* \$300.

2. If 25 tables cost \$400, what will 12 tables cost? *Ans.* \$192.

3. If 31 cows cost \$465. what will 60 cows cost? *Ans.* \$900.

4. If 43 readers cost 688 cents, what will 25 readers cost? *Ans.* 400 cents.

5. If 752 sheep cost \$4512, what will 137 sheep cost? *Ans.* \$822.

6. If 20 *bushels* = 640 *quarts*, then 17 *bushels* = ? *Ans.* 544 quarts.

7. If 15 *yards* = 540 *inches*, then 23 *yards* = ? *Ans.* 828 inches.

8. If 75 *ones* = 675 *ninths*, then 63 *ones* = ? *Ans.* 567 ninths.

143. Parallel Problems.

1.m What number multiplied by 12 will make 108?

2. One of the factors of 4375 is 175, what is the **c** factor? *Ans.* 25.

3.m If 9 boys together catch 72 fishes, how many does each boy catch on an average?

4. A field of 57 acres produced 1539 bushels of wheat, what was the average product of an acre? *Ans.* 27 bushels.

5.m If 11 yards of cloth cost 132 cents, what is the cost of 1 yard?

6. A man sold 59 acres of land for \$1062; what did he receive per acre?

7.m What number is contained in 68 9 times and 5 over?

8. A man had \$937, and, after paying some laborers at the rate of \$25 apiece, had \$12 left; how many laborers were there? *Ans.* 37.

9.m How often is 13 **a** less 5 **a** contained in 56 **a**?

10. A clerk's yearly salary is \$1500 and his expenses \$945; in how many years can he lay up \$4440? *Ans.* 8 years.

11.m A boy sold a merchant 4 oranges at 6 cents apiece, and 9 apples at 4 cents apiece, and took his pay in cigars at 5 cents a piece; how many cigars did he get? *Ans.* 12 cigars.

12. A farmer sold 27 cords of wood at $5 a cord, and 47 hogs at $7 apiece, and took in exchange flour at $8 a barrel; how many barrels did he get?

Ans. 58 barrels.

13.m How many minutes will it take 2 boys to remove 80 rails if one boy removes 6 rails, and the other 4 rails, every minute?

14. Two railway trains are 540 miles apart, and travel towards each other at the rates of 25 miles and 20 miles per hour; how many hours before they will meet? *Ans.* 12 hours.

15.m If a horse walks 35 miles in 7 hours, how far will he walk in 11 hours at the same rate?

16. If a railway train goes 304 miles in 16 hours, how far will it travel in 21 hours at the same rate?

Ans. 399 miles.

17.m How much is $\frac{1}{2}$ of the sum of 12 and 8?

18. What is the average width of a field which is 332 yards wide at one end, and 478 yards wide at the other? *Ans.* 405 yards.

19.m How much is $\frac{1}{3}$ of the sum of 11, 9, and 4?

20. A farmer killed 3 hogs; one weighed 165 pounds, another 173 pounds, and the third 181 pounds; what was the average weight of the three hogs?

Ans. 173 pounds.

21.m Henry started on a journey of 62 miles, and traveled at the rate of 5 miles a day for 4 days; how many days will it take him to complete the journey if he goes at the rate of 6 miles a day?

22. A laborer engaged to remove 4703 bricks. After

working 24 hours, removing 125 bricks each hour, how many hours will it take him to complete the work if he removes 131 bricks per hour? *Ans.* 13.

23.m Frank sold 10 oranges at 6 cents apiece, and with the money bought apples at 5 cents apiece; how many apples did he get?

24. A man sold 144 horses at $135 apiece, and invested the money in land at $18 per acre; how many acres did he get? *Ans.* 1080 acres.

25.m A boy spent 42 cents for cakes, and 1-*sixth* as much for nuts; what are the **c** parts of what he spent in all? How much did he spend?

26. A farmer has 306 acres in one field, and 1-*eighteenth* as much in another field; how much has he in both fields? *Ans.* 323 acres.

144. Questions for Review.

What is: 1. Division? 2. The dividend? 3. The divisor? 4. The qnotient? 5. The sign of division?

What is denoted by the sign $\div$?

Name the three principles of division. Can division be effected by subtraction?

What is the relation of division to multiplication? What do we find by division?

What is meant by: 1. Short division? 2. Long division? Give the rule for each.

How do we find: 1. The half of a number? 2. The *third* of a number? 3. The *fourth?* 4. The *fifth?* etc.

Divisors and Multiples.

145. A Divisor of a number is one of its factors.*

Thus, 4 is a divisor of 12, since $4 \times 3 = 12$. Also, 1, 2, 3, 4, 6, and 12 are divisors of 12, since each is contained in 12 an exact number of times.

1°. 1 is a divisor of every number.

2°. Every number is a divisor of itself.

Any number is exactly divisible:

3°. By 2, if its last figure is divisible by 2.

4°. By 4, if its last two figures are divisible by 4.

5°. By 8, if its last three figures are divisible by 8.

6°. By 3 or 9, if the sum of its figures is divisible by 3 or 9.

7°. By 5, if it ends in 5 or 0.

8°. By 6, if it is divisible by 2 and 3.

Mental Exercises.

How many times is:

1. 1 contained in 11? 1 contained in 143?

2. 13 contained in 13? 97 contained in 97?

Which of the numbers 2, 3, 4, 5, 6, 8, 9, and 10 are divisors of: 250? 3056? 4581? 1722? 45460? 761? 23202? 45128? 37301? 371820?

* The terms numbers, divisors, multiples, and factors, as here used, denote integers.

146. A Common Divisor of two or more numbers is any number which will exactly divide all of them.

Thus, 2 and 4 are common divisors of 8 and 12, since they divide each of them without a remainder.

Exercises.

1. What are the common divisors of 6, 12, and 24?
Ans. 2, 3, and 6.
2. What are the common divisors of 8, 16, and 24?
Ans. 2, 4, and 8.
3. What are the common divisors of 12, 18, and 24?
Ans. 2, 3, and 6.
4. What are the common divisors of 36, 54, and 72?
Ans. 2, 3, 6, 9, and 18.
5. What are the common divisors of 24, 36, and 48?
Ans. 2, 3, 4, 6, and 12.

147. The Greatest Common Divisor of two or more numbers, denoted by G. C. D., is the greatest number that will exactly divide each of them.

Answer these five questions by referring to the preceding exercises: What is the G. C. D. of 6, 12, 24? Of 8, 16, 24? Of 12, 18, 24? Of 36, 54, 72? Of 24, 36, 48?

148. Principle.—The **c** factors of the G. C. D. of several numbers are all the factors common to all.

Thus, the **c** factors of 12 are 2, 2, 3,
the **c** factors of 18 are 2, 3, 3.

Now, from each we may take the factor 2, then the factor 3, and as no equal factors remain, 2 and 3 are the **c** factors of the G. C. D. of 12 and 18.

149. What is the G. C. D. of 18, 27, and 36?

OPERATION.

3)18. 27. 36.
3) 6. 9. 12.
2. 3. 4.

EXPLANATION.—First divide 18, 27, and 36 by any number that will divide each of them, as 3. Then divide the quotients 6, 9, and 12, by any number that will exactly divide each of them, as 3. Now there is no number except 1 that will divide the quotients 2, 3, 4. Hence, 3 and 3, the two divisors, are the **c** factors of the G. C. D.; that is, the G. C. D. is 9.

WRITTEN EXERCISES.

1. What is the G. C. D. of 12 and 30? *Ans.* 6.
2. What is the G. C. D. of 24, 30, and 54? *Ans.* 6.
3. What is the G. C. D. of 35, 56, and 70? *Ans.* 7.
4. What is the G. C. D. of 18, 36, and 72? *Ans.* 18.
5. What is the G. C. D. of 48, 72, and 144? *Ans.* 24.
6. What is the G. C. D. of 32, 48, 64, and 160? *Ans.* 16.
7. What is the greatest common divisor of 60, 90, 150, and 210? *Ans.* 30.

150. A Prime Number is a number which has no divisors except itself and 1.

Thus, 1, 2, 3, 5, 7, etc, are prime numbers.

151. A Composite Number is a number which has other divisors than itself and 1.

Thus, 4, 6, 8, 9, 10, etc., are composite numbers.

Is 13 a prime or a composite number?

Ans. A prime number, as it has no divisors except 1 and 13.

Is 21 a prime or a composite number?

Ans. A composite number, as it can be divided by 3 and also by 7.

152. Prime Factors are factors which are prime numbers.

Thus, 2, 2, and 3 are the prime factors of 12.

Written Exercises.

153. 1. Write all the prime numbers between 1 and 25; 25 and 50; 50 and 75; 75 and 100.

2. What are the prime factors of 56?

OPERATION.

$$\begin{array}{r} 2)56 \\ 2)28 \\ 2)14 \\ 7)\ 7 \\ \hline 1 \end{array}$$

EXPLANATION.—We divide the number by any prime factor; then divide the quotient by any prime factor, etc., until the quotient 1 is obtained. The several divisors are the prime factors required. Hence, the answer is 2, 2, 2 and 7.

What are the prime factors of:

3. 50?	*Ans.* 2, 5. 5.	8. 680?	*Ans.* ?
4. 60?	*Ans.* 2, 2, 3, 5.	9. 1155?	*Ans.* ?
5. 108?	*Ans.* 2, 2, 3, 3, 2.	10. 7800?	*Ans.* ?
6. 640?	*Ans.* 5, *seven* 2's.	11. 2310?	*Ans.* ?
7. 455?	*Ans.* 5, 7, 13.	12. 4290?	*Ans.* ?

154. A Multiple of a number is a number which contains it an exact number of times.

Thus, 24 contains 6 exactly 4 times, hence 24 is a multiple of 6.

Is 16 a multiple of 8? 25 a multiple of 5? 27 of 7? 36 of 9? 42 of 6? 54 of 9? 63 of 7? 64 of 10? 72 of 12?

155. A Common Multiple of two or more numbers is a number which is exactly divisible by each of them.

Thus, 12 is a common multiple of 2, 3, 4, and 6; 18 of 2, 3, 6, and 9.

Exercises.

1. Name three common multiples of 2, 3, and 6.
Ans. 6, 12, and 18.
2. Name three common multiples of 4, 5, and 10.
Ans. 20, 40, 60.
3. Name three common multiples of 3, 4, 12.
Ans. 12, 24, 36.
4. Name three common multiples of 7 and 2; 5 and 8; 3, 7, and 2; 5 and 12; 3, 5, and 10; 2, 4, and 8.

156. The Least Common Multiple of two or more numbers, denoted by L. C. M., is the smallest number which these numbers will exactly divide.

Answer these questions by referring to the preceding exercises: What is the L. C. M. of 2, 3, and 6? Of 4, 5, and 10? Of 3, 4, and 12?

What is the L. C. M. of 5 and 6? 6 and 7? 6 and 8? 5 and 9? 7 and 10? 2, 3, and 5?

157. Principle.—The **c** factors of the L. C. M. of two or more numbers are all the prime factors of each.

Thus, the prime factors of 12 are 2, 2, and 3.
" " 30 are 2, 3, and 5.
" " 50 are 2, 5, and 5.

Taking out the facto. 2 from each, then the factor 3 from the first and second, then the factor 5 from the second and third, there are left, 2 in the first, and 5 in the third. Hence, the **c** factors of the L. C. M. of 12, 30, and 50 are 2, 3, 5, 2, and 5.

Written Exercises.

158. What is the L. C. M. of 12, 15, and 25?

Explanation.—Divide any two or more of the numbers by any common prime factor, as 3, and bring down with the quo-

tients 4 and 5 such numbers (25) as do not contain the divisor. Again, divide out by 5, as it is a prime factor common to 5 and 25, and bring down the 4. Now there is no factor except 1 that will divide two of the numbers 4, 1, and 5. Hence, the two divisors 3 and 5, and the quotients 4 and 5 are the **c** factors of the L. C. M.; that is, the L. C. M. is $3 \times 5 \times 4 \times 5 = 300$, as it contains all the prime factors of 12, 15, and 25.

OPERATION.

$$\begin{array}{r|lll} 3) & 12 & 15 & 25 \\ \hline 5) & 4 & 5 & 25 \\ \hline & 4 & 1 & 5 \end{array}$$

Find the L. C. M. of:

2. 6 and 12; 9 and 15; 20 and 25. *Ans.* 12; 45; 100.
3. 12 and 20; 16 and 24; 35 and 42. *Ans.* 60; 48; 210.
4. 2, 4, 5, and 12; 3, 7, 9, and 14. *Ans.* 60; 126.
5. 5, 8, 10, and 12; 7, 9, 12, and 18. *Ans.* 120; 252.
6. 5, 6, 10, and 15; 6, 12, 15, and 20. *Ans.* 30; 60.
7. 15, 20, 30, and 40; 16, 20, 32, and 40. *Ans.* 120; 160.
8. 25, 36, 50, and 72; 48, 60, 96, and 120. *Ans.* 1800; 480.
9. 5, 7, 11, and 15; 3, 7, 13, and 39. *Ans.* 1155; 273.
10. 4, 5, 6, 10, 12, 15, 20, and 30. *Ans.* 60.

159. Questions for Review.

What is: 1. A divisor of a number? 2. A common divisor of two or more numbers? 3. The G. C. D. of two or more numbers? Give an example of each.

What is: 1. A prime number? 2. A composite number? 3. A prime factor? 4. A multiple of a number? 5. A common multiple of two or more numbers? 6. The L. C. M. of two or more numbers? Give an example of each.

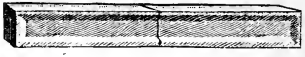
What is the principle of: 1. The G. C. D.? 2. The L. C. M.?

When is a number exactly divisible by: 2? 3? 4? 5? 6? 8? 9? 10?

Common Fractions.

Inductive Exercises.

160. When an apple, an orange, a number, or a bar of soap is divided into two equal parts, what is each part called? How is 1-half written? *Ans.* $\frac{1}{2}$.

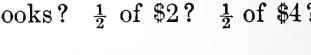


How do we get $\frac{1}{2}$ of a thing?

Ans. Divide it into two equal parts, and take one of the parts.

What is $\frac{1}{2}$ of a pile of 2 books? $\frac{1}{2}$ of \$2? $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of \$16?

When an apple, an orange, a number, or a bar of soap is divided into three equal parts, what is each part called? What are two of the parts called? How are 1-third and 2-thirds written? *Ans.* $\frac{1}{3}$ and $\frac{2}{3}$.



How do we get $\frac{1}{3}$ of a thing?

Ans. Divide it into three equal parts and take one of the parts.

What is $\frac{1}{3}$ of a pile of 3 books? $\frac{1}{3}$ of \$3? $\frac{1}{3}$ of \$6? $\frac{1}{3}$ of \$12? $\frac{1}{3}$ of 24c.? $\frac{1}{3}$ of 60 bushels?

How do you get $\frac{2}{3}$ of a thing?

Ans. Divide it into three equal parts and take one of the parts 2 times.

What is $\frac{1}{3}$ of a pile of 3 books? $\frac{1}{3}$ of \$3? $\frac{1}{3}$ of \$12? $\frac{1}{3}$ of 27c.?

When an apple, an orange, a number or a bar of soap is divided into four equal parts, what is each part called? What are two of the parts called? What are three of the parts called? How are 1-fourth, 2-fourths, and 3-fourths written?

Ans. $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$.

How do we get $\frac{1}{4}$ of a thing?

Ans. Divide it into four equal parts, and take one of the parts.

What is $\frac{1}{4}$ of a pile of 4 books? $\frac{1}{4}$ of \$4? $\frac{1}{4}$ of \$20? $\frac{1}{4}$ of 40c.?

How do we get $\frac{2}{4}$ of a thing?

Ans. Divide it into four equal parts and take 1 part 2 times.

How much is $\frac{2}{4}$ of a pile of 4 books? $\frac{2}{4}$ of 4? $\frac{2}{4}$ of 16? $\frac{2}{4}$ of 32?

How do we get $\frac{3}{4}$ of a thing?

Ans. Divide it into four equal parts, and take 1 part 3 times.

How much is $\frac{3}{4}$ of a pile of 4 books? $\frac{3}{4}$ of 4? $\frac{3}{4}$ of 12? $\frac{3}{4}$ of 40?

Definitions.

161. Fractional Parts are parts obtained by dividing any thing, or a *unit*, into any number of equal parts.

Thus: halves, thirds, fourths, fifths, sevenths, tenths, etc., are fractional parts.

162. A Fractional Unit is one of the equal fractional parts into which a thing is divided.

Thus: { 1-half, 1-third, 1-fourth, 1-fifth, 1-tenth, 1-twelfth, } are fractional units.
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{12}$

163. A Fraction is a fractional unit taken one or more times.

Thus: { 1-fourth, 2-fifths, 3-sevenths, 7-tenths, } are fractions.
$1 \times \frac{1}{4}$ $2 \times \frac{1}{5}$ $3 \times \frac{1}{7}$ $7 \times \frac{1}{10}$
$\frac{1}{4}$ $\frac{2}{5}$ $\frac{3}{7}$ $\frac{7}{10}$

164. The Terms of a fraction are the two numbers used to express it.

165. The Denominator is that term which *names* the parts expressed by the fraction. It is written below the horizontal line.

166. The Numerator is that term which *numbers* the parts expressed by the fraction. It is written above the horizontal line.

Thus: the terms of the fraction $\frac{5}{6}$ are 5 and 6; the denominator is 6, and the numerator is 5.

The fraction $\frac{5}{6}$ means:

1°. 5 of the parts when 1 unit has been divided into 6 equal parts.

2°. The fractional unit 1-sixth, or $\frac{1}{6}$, taken 5 times.

3°. The quotient of 5 divided by 6.

Read each of the following fractions, name the terms, the numerator, the denominator, the fractional unit, the number of fractional units, and give the three meanings of each fraction:

$\frac{2}{3}$; $\frac{3}{4}$; $\frac{7}{8}$; $\frac{9}{10}$; $\frac{7}{5}$; $\frac{13}{6}$; 5 *ninths*; 7 *twelfths*.

167. A Proper Fraction is one of which the numerator is less than the denominator; as $\frac{5}{7}$, $\frac{3}{8}$, $\frac{5}{9}$, etc.

168. An Improper Fraction is one of which the numerator equals or exceeds the denominator; as $\frac{5}{5}$, $\frac{10}{9}$, $\frac{25}{6}$, etc.

169. A Mixed Number is an expression consisting of a whole number and a fraction; as $2\frac{1}{3}$, $5\frac{3}{4}$, $12\frac{1}{2}$, etc.

170. The Value of a fraction is the quotient of the numerator divided by the denominator.

The value of $\frac{8}{4}$ is 2; of $\frac{12}{3}$ is 4; of $\frac{35}{7}$ is 5.

When the numerator equals the denominator the value of the fraction equals 1; as $\frac{4}{4}$, $\frac{7}{7}$, $\frac{13}{13}$, etc.

The value of a proper fraction is less than 1, as its numerator is less than its denominator.

The value of an improper fraction is equal to or exceeds 1, as its numerator equals or exceeds its denominator.

171. A Compound Fraction is a fraction of a fraction; as $\frac{1}{3}$ of $\frac{1}{4}$, $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{5}{11}$.

Mental Exercises.

172. 1. How many halves ($\frac{1}{2}$) in *one* (1)? Why?

2. What is the worth of *one* orange, if 1 *half* of it is worth 5 cents? 6c.? 10c.?

3. How many *thirds* ($\frac{1}{3}$) in *one* (1)? Why?

4. What is the worth of *one* bale of cotton, if 1 *third* of it is worth \$10? \$12? \$20?

5. How many *fifths* ($\frac{1}{5}$) in *one* (1)? Why?

6. What is the length of a pole, if 1 *fifth* of it is 3 feet long? 7 feet long? 10 feet?

7. How many *sevenths* ($\frac{1}{7}$) in *one* (1)? Why?

8. What is the weight of a rock if 1 *seventh* of it weighs 5 pounds? 9 pounds?

9. How many *tenths* in *one* (1)? Why?

10. How many marbles in a box, if 1 *tenth* of them is 6 marbles? 9 marbles? 25 marbles?

11. What is the number whose *half* is 1? Whose *third* is 2? Whose *fourth* is 5? Whose *sixth* is 2? Whose *ninth* is 3? Whose *tenth* is 7?

12. How much is 3-*fourths* ($\frac{3}{4}$) of 28?

ANALYSIS.—1 *fourth* of 28 is 7; 3 *fourths* of 28 is 3 times 7 = 21, *Ans.*

13. How much is 2 *thirds* of 12? $\frac{2}{3}$ of 15? $\frac{2}{3}$ of 30?

14. How much is 3 *fourths* of 16? $\frac{3}{4}$ of 20? $\frac{3}{4}$ of 40?

15. How much is 4 *fifths* of 20? $\frac{3}{5}$ of 30? $\frac{2}{5}$ of \$50?

16. How much is 5 *sixths* of 18? $\frac{3}{6}$ of 24? $\frac{2}{6}$ of 60c.?

17. How much is 7 *ninths* of 18? $\frac{5}{9}$ of 36? $\frac{2}{9}$ of 45c.?

18. How much is 5 *twelfths* of 24? $\frac{11}{12}$ of 36? $\frac{7}{12}$ of \$60.

19. If *one* acre of land cost \$12, what is the cost of $\frac{1}{3}$ of an acre? $\frac{1}{6}$ of an acre? $\frac{3}{4}$ of an acre?

20. If *one* bushel of potatoes cost 30 cents, what is the cost of $\frac{1}{5}$ of a bushel? $\frac{3}{5}$ of a bushel? $\frac{7}{10}$ of a bushel?

21. 5 *sixths* of a number is 10, what is the number?

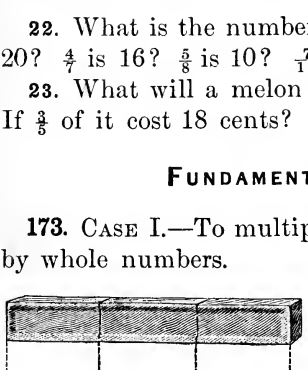
ANALYSIS.—If 5 *sixths* is 10, 1 *sixth* is $\frac{1}{5}$ of 10, or 2; hence, the number is 6 times 2 = 12, *Ans.*

22. What is the number of which 5 *sixths* is 20? $\frac{2}{6}$ is 20? $\frac{4}{7}$ is 16? $\frac{5}{8}$ is 10? $\frac{7}{12}$ is 21?

23. What will a melon cost if $\frac{3}{4}$ of it cost 20 cents? If $\frac{3}{5}$ of it cost 18 cents? If $\frac{5}{8}$ of it cost 35 cents?

FUNDAMENTAL PRINCIPLES.

173. CASE I.—To multiply and divide fractional units by whole numbers.



Into how many parts is this bar of soap divided? What is 1 part called? Is it a fractional unit?

If each of the three parts is cut into two parts, how many parts will there be in all? What is 1 part called? Is it a fractional unit

Is 1 part of the first bar equal to 2 times 1 part of the second bar?

What does this show? *Ans.* That 2 *times* $\frac{1}{6} = \frac{1}{3}$.

What else does it show? *Ans.* That 1 *half* of $\frac{1}{3} = \frac{1}{6}$.

Hence,

1°. *To multiply a fractional unit by a number, we may divide the denominator by that number.*

2°. *To divide a fractional unit by a number, we multiply the denominator by that number.*

Mental Exercises.

How much is:

1. 2 times $\frac{1}{8}$? *Ans.* $\frac{1}{4}$.
2. 4 times $\frac{1}{8}$? *Ans.* $\frac{1}{2}$.
3. 5 times $\frac{1}{15}$? *Ans.* $\frac{1}{3}$.
4. $2 \times \frac{1}{12}$?
5. $7 \times \frac{1}{21}$?
6. $11 \times \frac{1}{66}$?
7. 5×1 *tenth?*
8. 8×1 *fortieth?*
9. 3×1 *fifteenth?*

How much is:

10. 1 *half* of $\frac{1}{3}$? *Ans.* $\frac{1}{6}$.
11. 1 *fourth* of $\frac{1}{2}$? *Ans.* $\frac{1}{8}$.
12. 1 *sixth* of $\frac{1}{3}$? *Ans.* $\frac{1}{18}$.
13. 1 *fifth* of $\frac{1}{7}$? *Ans.* $\frac{1}{35}$.
14. 1 *fourth* $\div 3$?
15. 1 *tenth* $\div 5$?
16. 1 *third* $\div 7$?
17. 1 *ninth* $\div 10$?

174. Case II.—To change the fractional unit.

Into how many equal parts is this bar of soap divided? What is 1 part called? 2 parts? Is $\frac{1}{3}$ a fractional unit?

Here is an equal bar; into how many parts is it divided? What is 1 part called? 2 parts? etc. Is $\frac{1}{6}$ a fractional unit?

Is 1 part of the first bar, or $\frac{1}{3}$, equal to two parts of the second, or $\frac{2}{6}$?

Are 2 parts of the first bar equal to 4 parts of the second?

What does this show? *Ans.* That $\frac{2}{3} = \frac{4}{6}$, and $\frac{4}{6} = \frac{2}{3}$.

Hence,

1°. *Multiplying both terms of a fraction by the same number does not alter the value of the fraction.*

2°. *Dividing both terms of a fraction by the same number does not alter the value of the fraction.*

Explanation.—Can 24 be written thus: 6 *fours?* Thus: 12 *twos?* Are 6 *fours* equal to 12 *twos?* Why? *Ans.* The unit *two* is *half* of the unit *four*, but is taken *twice* as often, since 12 is twice 6; hence, they are equal.

Are 3 *fifths*, or $\frac{3}{5}$, equal to 6 *tenths*, or $\frac{6}{10}$? Why? *Ans.* The unit 1 *tenth* is *half* of the unit 1 *fifth*, but is taken *twice* as often, since 6 is twice 3; hence, $\frac{3}{5} = \frac{6}{10}$.

Why, then, is the value of a fraction not changed by multiplying both terms by the same number? *Ans.* Because it decreases the fractional unit in the same ratio that it increases the number of times it is taken.

Why is the value of a fraction not changed by dividing both terms by the same number? *Ans.* Because it increases the fractional unit in the same ratio that it decreases the number of times it is taken.

Reduction of Fractions.

175. Reduction of a Fraction consists in changing its terms without altering its value.

176. Case I.—To reduce a fraction to its lowest terms.

A fraction is in the lowest terms when no number greater than 1 will exactly divide its numerator and denominator.

1. Reduce $\frac{30}{40}$ to its lowest terms.

OPERATION.

$2)\frac{30}{40} = \frac{15}{20}$

$5)\frac{15}{20} = \frac{3}{4}$

EXPLANATION.—Dividing both terms of $\frac{30}{40}$ by 2, we get $\frac{15}{20}$. Now dividing both terms of $\frac{15}{20}$ by 5, we obtain $\frac{3}{4}$.

In the second operation we divide both terms by their G. C. D., 10.

2d OPERATION.

$10)\frac{30}{40} = \frac{3}{4}$

Hence the,

RULE.—*Divide both terms of the fraction by any number greater than* 1 *that will exactly divide them, and continue the operation as long as possible.*

Or, *Divide both terms by their G. C. D.*

WRITTEN EXERCISES.

Reduce to lowest terms:

2. $\frac{14}{35}$, $\frac{21}{56}$, $\frac{24}{36}$, $\frac{45}{63}$, $\frac{75}{125}$. *Ans.* $\frac{2}{5}$, $\frac{3}{8}$, $\frac{2}{3}$, ? ?

3. $\frac{84}{196}$, $\frac{112}{140}$, $\frac{81}{108}$, $\frac{63}{81}$, $\frac{135}{140}$. *Ans.* $\frac{3}{7}$, $\frac{4}{5}$, $\frac{3}{4}$, ? ?

4. $\frac{207}{99}$, $\frac{128}{88}$, $\frac{39}{143}$, $\frac{195}{225}$, $\frac{528}{1012}$. *Ans.* $\frac{23}{11}$, ? $\frac{3}{11}$, ? $\frac{12}{23}$.

177. CASE II.—To reduce a whole number to a fraction.

1. Reduce 4 to a fraction whose denominator is 3.

EXPLANATION.—We write 4 thus: $\frac{4}{1}$, then multiply both terms by 3, and obtain $\frac{12}{3}$.

OPERATION.

$\frac{4}{1} = \frac{4\times3}{1\times3} = \frac{12}{3}$.

Hence, the

RULE.—*Multiply the whole number by the given denominator and place the product over the denominator.*

WRITTEN EXERCISES.

Reduce:

2. 5 to a fraction whose denominator is 3. *Ans.* $\frac{15}{3}$.

3. 7 to a fraction whose denominator is 8. *Ans.* $\frac{56}{8}$.

4. 9 to a fraction whose denominator is 6. *Ans.* $\frac{54}{6}$.
5. 11 to a fraction whose denominator is 5. *Ans.* ?

Reduce:

6. 7 to *fifths.* *Ans.* $\frac{35}{5}$.
7. 8 to *thirds.* *Ans.* $\frac{24}{3}$.
8. 15 to *fourths.* *Ans.* ?
9. 20 to *tenths.* *Ans.* ?

178. CASE III.—To reduce a fraction to higher terms.

1. Reduce $\frac{3}{5}$ to a fraction whose denominator is 15.

EXPLANATION.—Write $\frac{3}{5}$, and on its right place 15 with a line above it. Now say 5 in 15 3 times. $3 \times 3 = 9$, which place over the 15.

OPERATION.

$\frac{3}{5} = \frac{9}{15}$

Hence, the

RULE.—*Divide the denominator of the required fraction by the denominator of the given fraction, multiply the quotient by the numerator, and place the product over the larger denominator.*

WRITTEN EXERCISES.

Reduce:

2. $\frac{2}{3}$ to a fraction whose denom. is 24. *Ans.* $\frac{16}{24}$.
3. $\frac{5}{6}$ to a fraction whose denom. is 42. *Ans.* $\frac{35}{42}$.
4. $\frac{7}{11}$ to a fraction whose denom. is 66. *Ans.* ?
5. $\frac{3}{4}$ to *sixteenths.* *Ans.* $\frac{12}{16}$.
6. $\frac{5}{9}$ to *forty-fifths.* *Ans.* $\frac{25}{45}$.
7. $\frac{5}{8}$ to *fortieths.* *Ans.* ?
8. $\frac{7}{15}$ to *sixtieths.* *Ans.* ?

179. CASE IV.—To reduce a mixed number to an improper fraction.

1. Reduce $5\frac{2}{3}$ to an improper fraction.

EXPLANATION.—We multiply 5 by 3, add 2 to the product, and place the sum 17 over the denominator.

OPERATION.

$5 \times 3 = 15$

$15 + 2 = 17$, $\frac{17}{3}$.

ANALYSIS.—$5\frac{2}{3} = 5$ *ones* and 2 *thirds*,
$= 15$ *thirds* and 2 *thirds* $= 17$ *thirds*.

Hence, the

RULE.—*Multiply the whole number by the denominator, add the numerator to the product, and place the sum over the denominator.*

WRITTEN EXERCISES.

Reduce to improper fractions:

2. $4\frac{1}{3}$.	*Ans.* $\frac{13}{3}$.	**8.** $12\frac{1}{3}$.	*Ans.* $\frac{37}{3}$.	**14.** $34\frac{7}{10}$.	*Ans.* ?
3. $7\frac{2}{4}$.	*Ans.* $\frac{30}{4}$.	**9.** $15\frac{3}{4}$.	*Ans.* $\frac{63}{4}$.	**15.** $45\frac{3}{11}$.	*Ans.* ?
4. $5\frac{3}{5}$.	*Ans.* $\frac{28}{5}$.	**10.** $16\frac{2}{5}$.	*Ans.* $\frac{50}{3}$.	**16.** $35\frac{11}{12}$.	*Ans.* ?
5. $6\frac{2}{9}$.	*Ans.* ?	**11.** $18\frac{3}{5}$.	*Ans.* ?	**17.** $60\frac{17}{20}$.	*Ans.* ?
6. $10\frac{3}{4}$.	*Ans.* ?	**12.** $17\frac{4}{9}$.	*Ans.* ?	**18.** $135\frac{23}{30}$.	*Ans.* ?
7. $9\frac{3}{5}$.	*Ans.* ?	**13.** $21\frac{5}{6}$.	*Ans.* ?	**19.** $244\frac{17}{45}$.	*Ans.* ?

180. CASE V.—To reduce an improper fraction to a whole or mixed number.

1. Reduce $\frac{10}{3}$ to a mixed number.

OPERATION.

$$3)10(3\tfrac{1}{3}$$
$$\underline{9}$$
$$1$$

EXPLANATION.—We divide the numerator by the denominator.

ANALYSIS.—$\frac{10}{3} = 10$ *thirds* $= 9$ *thirds* and 1 *third*
$= 3$ and 1 *third* $= 3\frac{1}{3}$.

2. Reduce $\frac{22}{4}$ to a mixed number.

OPERATION.

$$4)22(5\tfrac{1}{2}$$
$$\underline{20}$$
$$2)\tfrac{2}{4} = \tfrac{1}{2}.$$

ANALYSIS.—$\frac{22}{4} = 22$ *fourths*
$= 20$ *fourths and* 2 *fourths* $= 5$ and $\frac{1}{2}$.

Hence, the

RULE.—*Divide the numerator by the denominator, and reduce the fractional remainder, if any, to its lowest terms.*

Written Exercises.

Reduce to whole or mixed numbers:

3. $\frac{17}{4}$.	*Ans.* $4\frac{1}{4}$.	8. $\frac{64}{4}$.	*Ans.* 16.	13. $\frac{200}{15}$.	*Ans.* $13\frac{1}{3}$.
4. $\frac{27}{3}$.	*Ans.* 9.	9. $\frac{85}{10}$.	*Ans.* $8\frac{1}{2}$.	14. $\frac{625}{40}$.	*Ans.* ?
5. $\frac{45}{6}$.	*Ans.* $7\frac{1}{2}$.	10. $\frac{96}{9}$.	*Ans.* ?	15. $\frac{256}{72}$.	*Ans.* ?
6. $\frac{63}{12}$.	*Ans.* $5\frac{1}{4}$.	11. $\frac{100}{8}$.	*Ans.* ?	16. $\frac{512}{96}$.	*Ans.* $5\frac{1}{3}$.
7. $\frac{48}{9}$.	*Ans.* $5\frac{1}{3}$.	12. $\frac{78}{12}$.	*Ans.* ?	17. $\frac{629}{51}$.	*Ans.* $12\frac{1}{3}$.

181. Case VI.—To reduce a compound fraction to a simple one.

1. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

Analysis.—By Art. **173**, $\frac{1}{5} = \frac{1}{15}$; then $\frac{2}{3}$ of $\frac{1}{5}$ will be 2 times 1 fifteenth, or $\frac{2}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ will be 4 *times* 2 fifteenths, or $\frac{8}{15}$, *Ans.* Hence,

operation.

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Rule.—*Multiply the numerators together for the numerator of the answer, and the denominators together for the denominator.*

Exercises.

2. Reduce $\frac{3}{4}$ of $\frac{6}{7}$ to a simple fraction. *Ans.* $\frac{18}{28} = \frac{9}{14}$.
3. Change $\frac{5}{9}$ of $\frac{3}{10}$ to a simple fraction. *Ans.* $\frac{1}{6}$.
4. Reduce 2 *thirds* of 3 *tenths* to a simple fraction. *Ans.* 1 *fifth*.
5. Change 4 *fifths* of 2 *sevenths* to a simple fraction. *Ans.* $\frac{8}{35}$.
6. What simple fraction is equal to $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{3}{4}$? *Ans.* $\frac{5}{14}$.
7. What simple fraction is equal to $\frac{3}{7}$ of $\frac{5}{9}$ of $\frac{12}{15}$. *Ans.* $\frac{4}{21}$.
8. Reduce $\frac{3}{4}$ of $7\frac{1}{3}$ of 5 to a simple fraction.

EXPLANATION.—Reduce $7\frac{1}{3}$ to an improper fraction ($\frac{22}{3}$); also the whole number 5, by putting 1 under it for a denominator, and proceed as before.

OPERATION.

$$\frac{3}{4} \times \frac{22}{3} \times \frac{5}{1} = \frac{330}{12} = \frac{55}{2}.$$

9. Reduce $\frac{5}{8}$ of $3\frac{1}{5}$ of $\frac{1}{4}$ to a simple fraction. *Ans.* $\frac{1}{2}$.
10. Reduce $\frac{1}{3}$ of $\frac{6}{7}$ of $9\frac{1}{3}$ to a simple fraction. *Ans.* $\frac{8}{3}$.
11. What is the value of $\frac{3}{4}$ of $\frac{5}{7}$ of $1\frac{13}{15}$? *Ans.* 1.
12. What is the value of $\frac{2}{3}$ of $\frac{3}{11}$ of $16\frac{1}{2}$? *Ans.* 3.

182. Cancellation. Instead of multiplying the numerators, then the denominators, and then reducing to lowest terms, the same result may be obtained by first striking out or *cancelling* all factors common to the numerator and denominator. By this process the work is often materially shortened.

13. Reduce $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{7}$ to a simple fraction.

EXPLANATION.—First, there is a 3 in both numerator and denominator, *one* over 5, and the *other* under 2; we cancel these by making a mark across them. Next, we cancel the two 5's in the same manner. Cancelling a number is dividing by it; hence, 1 is supposed to take the place of the number canceled; that is, $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{7}$ *means* $\frac{1}{1}$ of $\frac{2}{1}$ of $\frac{1}{7}$. Hence the answer is $\frac{2}{7}$.

OPERATION.

$$\frac{\cancel{3}}{\cancel{5}} \text{ of } \frac{2}{\cancel{3}} \text{ of } \frac{\cancel{5}}{7} = \frac{2}{7}$$

14. Reduce $\frac{2}{9}$ of $\frac{3}{5}$ of $\frac{15}{2}$ to a simple fraction.

EXPLANATION. — Write 3×3 in the place of 9, 3×5 in the place of 15; then cancel as in the preceding example.

OPERATION.

$$\frac{\cancel{2}}{\cancel{3} \times \cancel{3}} \text{ of } \frac{\cancel{3}}{\cancel{5}} \text{ of } \frac{\cancel{3} \times \cancel{5}}{\cancel{2}} = 1.$$

15. Reduce $\frac{3}{5}$ of $\frac{7}{6}$ of $\frac{15}{11}$ to a simple fraction. *Ans.* $2\frac{1}{22}$.

Reduce to a simple fraction:

16. $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{4}$ of 6. *Ans.* 3.

17. $\frac{3}{7}$ of $\frac{5}{2}$ of $\frac{3}{8}$ of $\frac{56}{9}$ of $\frac{4}{5}$. *Ans.* 2.
18. $\frac{3}{4}$ of $\frac{2}{11}$ of $\frac{5}{7}$ of $7\frac{1}{3}$ of $\frac{7}{10}$. *Ans.* $\frac{1}{2}$.
19. $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{4}{9}$ of $\frac{6}{11}$ of $3\frac{3}{5}$ of $8\frac{5}{9}$ of $\frac{1}{2}$. *Ans.* 1.

183. CASE VII.—To reduce fractions to a common denominator.

Fractions have a common denominator when their denominators are the same: as $\frac{3}{7}$ and $\frac{4}{7}$, $\frac{5}{11}$ and $\frac{4}{11}$.

1. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

Multiplying both terms of $\frac{1}{2}$ by 3, the denominator of the other fraction, we have $\frac{1}{2}=\frac{3}{6}$. Now multiplying both terms of $\frac{1}{3}$ by 2, the denominator of the other fraction, we have $\frac{1}{3}=\frac{2}{6}$. Hence, in the place of $\frac{1}{2}$ and $\frac{1}{3}$ we have $\frac{3}{6}$ and $\frac{2}{6}$, and these have a common denominator.

Reduce to a common denominator:

2. $\frac{3}{4}$ and $\frac{5}{7}$. *Ans.* $\frac{21}{28}$ and $\frac{20}{28}$.
3. $\frac{2}{9}$ and $\frac{3}{4}$. *Ans.* $\frac{8}{36}$ and $\frac{27}{36}$.
4. $\frac{1}{2}$ and $\frac{5}{3}$. *Ans.* $\frac{3}{6}$ and $\frac{10}{6}$.
5. $\frac{1}{3}$ and $\frac{3}{5}$. *Ans.* ?
6. $\frac{7}{9}$ and $\frac{5}{8}$. *Ans.* ?
7. $\frac{3}{4}$ and $\frac{3}{11}$. *Ans.* ?

8. Reduce $\frac{4}{5}$, $\frac{2}{7}$, and $\frac{3}{8}$ to a common denominator.

EXPLANATION.—We multiply both terms of $\frac{4}{5}$ by 7×8, the denominators of the other fractions, which gives $\frac{4}{5}=\frac{224}{280}$. We next multiply both terms of $\frac{2}{7}$ by 5×8, the denominators of the other fractions, and obtain $\frac{2}{7}=\frac{80}{280}$. Similarly we get $\frac{3}{8}=\frac{105}{280}$. Hence, instead of the given fractions we have their equals: $\frac{224}{280}$, $\frac{80}{280}$, and $\frac{105}{280}$, which have a common denominator.

OPERATION.

$\frac{4}{5}\ 7\times8=\frac{224}{280}$.
$\frac{2}{7}\ 5\times8=\frac{80}{280}$.
$\frac{3}{8}\ 5\times7=\frac{105}{280}$.

Hence, the

RULE.—*Multiply both terms of each fraction by the product of all the denominators except its own.*

NOTE.—Mixed and whole numbers, if any, must be reduced to improper fractions.

Written Exercises.

Reduce to common denominators:

9. $\frac{5}{7}$ and $\frac{4}{5}$. *Ans.* $\frac{25}{35}$ and $\frac{28}{35}$.

10. $\frac{7}{10}$ and $\frac{3}{7}$. *Ans.* $\frac{49}{70}$ and $\frac{30}{70}$.

11. $\frac{4}{6}$ and $\frac{3}{5}$. *Ans.* ?

12. $\frac{7}{8}$ and $\frac{9}{13}$. *Ans.* $\frac{91}{104}$ and $\frac{72}{104}$.

13. $\frac{17}{20}$ and $\frac{11}{13}$. *Ans.* ?

14. $\frac{161}{31}$ and $\frac{11}{20}$. *Ans.* ?

15. $\frac{3}{4}$, $\frac{5}{7}$, and $\frac{1}{2}$. *Ans.* $\frac{42}{56}$, $\frac{40}{56}$, $\frac{28}{56}$.

16. $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$. *Ans.* $\frac{36}{60}$, $\frac{40}{60}$, $\frac{45}{60}$.

17. $\frac{5}{6}$, $\frac{4}{5}$, and $\frac{2}{3}$. *Ans.* $\frac{75}{90}$, $\frac{72}{90}$, $\frac{60}{90}$.

18. $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $\frac{140}{160}$, $\frac{120}{160}$, $\frac{128}{160}$.

19. $3\frac{1}{2}$, $\frac{2}{3}$, and $1\frac{1}{5}$. *Ans.* $\frac{105}{30}$, $\frac{20}{30}$, $\frac{36}{30}$.

20. 5, $1\frac{1}{2}$, and $\frac{3}{5}$. *Ans.* $\frac{50}{10}$, $\frac{15}{10}$, $\frac{6}{10}$.

21. $\frac{8}{9}$, $\frac{1}{2}$, and $\frac{2}{5}$. *Ans.* ?

22. $\frac{2}{3}$, 4, and $2\frac{1}{5}$. *Ans.* ?

23. $\frac{3}{7}$, $2\frac{1}{4}$, and 10. *Ans.* ?

184. To reduce fractions to their Least Common Denominator.

Rule.—*Find the L. C. M. of the denominators; take it for a common denominator, and reduce each fraction according to Case III.*

24. Reduce $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$ to their least common denominator.

Explanation.—The L. C. M. of 8, 6, and 12 is 24, which we take for a common denominator. Now by Case III we say, 8 in 24 3 times, $3 \times 3 = 9$; hence, $\frac{3}{8} = \frac{9}{24}$. In a similar manner we find $\frac{5}{6} = \frac{20}{24}$, and $\frac{7}{12} = \frac{14}{24}$.

operation.

$$\frac{3}{8} = \frac{9}{24}$$
$$\frac{5}{6} = \frac{20}{24}$$
$$\frac{7}{12} = \frac{14}{24}$$

Reduce to their least common denominators:

25. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$. *Ans.* $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

26. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{6}$, and $\frac{7}{18}$. *Ans.* $\frac{12}{18}$, $\frac{8}{18}$, $\frac{15}{18}$, $\frac{7}{18}$.

27. $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{32}$. *Ans.* $\frac{8}{32}$, $\frac{12}{32}$, $\frac{10}{32}$, $\frac{7}{32}$.

28. $\frac{3}{10}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{9}{20}$. *Ans.* $\frac{18}{60}$, $\frac{25}{60}$, $\frac{28}{60}$, $\frac{27}{60}$.

Addition of Fractions.

Inductive Exercises.

185. Parallelisms.

Whole Numbers	Fractions.
1. Add:	1. Add:
4 *threes* and 7 *threes*.	4 *thirds* and 7 *thirds*.
Ans. 11 *threes*.	*Ans.* 11 *thirds*.
2. Add:	2. Add:
8 *threes* and 9 *fours*.	8 *thirds* and 9 *fourths*.
These are unlike, and must be reduced to like units.	These are unlike, and must be reduced to like units.
8 *threes* = 2 *twelves*.	8 *thirds* = 32 *twelfths*.
9 *fours* = 3 *twelves*.	9 *fourths* = 27 *twelfths*.
Now adding the like numbers, we get	Now adding the like numbers, we get
5 *twelves*.	59 *twelfths*.

Principle.—*To add two numbers, whether whole or fractional, they must be reduced, if not already so, to like units.*

186. Case I.—When the denominators, or fractional units, are alike.

1. Add together $\frac{3}{7}$, $\frac{2}{7}$, and $\frac{1}{7}$.

Operation.

$\frac{3}{7} = 3$ *sevenths*.
$\frac{2}{7} = 2$ *sevenths*.
$\frac{1}{7} = 1$ *seventh*.
6 *sevenths* $= \frac{6}{7}$.

Explanation.—Since the numbers to be added have the same unit, viz: 1 *seventh*, we add as in whole numbers, and obtain 6 *sevenths*, or $\frac{6}{7}$. Hence,

Rule.—*Add the numerators and place the sum over the common denominator.*

What is the sum of:

2. $\frac{3}{5}$, $\frac{4}{5}$, $\frac{2}{5}$? *Ans.* $\frac{9}{5} = 1\frac{4}{5}$.
3. $\frac{3}{7}$, $\frac{4}{7}$, $\frac{6}{7}$? *Ans.* $1\frac{6}{7}$.
4. $\frac{4}{9}$, $\frac{5}{9}$, $\frac{1}{9}$, $\frac{8}{9}$? *Ans.* 2.
5. $\frac{7}{11}$, $\frac{2}{11}$, $\frac{5}{11}$, $\frac{4}{11}$? *Ans.* $1\frac{7}{11}$.
6. $\frac{3}{10}$, $\frac{7}{10}$, $\frac{1}{10}$, $\frac{4}{10}$? *Ans.* $1\frac{1}{2}$.
7. $\frac{5}{12}$, $\frac{11}{12}$, $\frac{1}{12}$, $\frac{7}{12}$? *Ans.* ?
8. $\frac{3}{8}$, $\frac{7}{8}$, $\frac{5}{8}$? *Ans.* ?
9. \$$\frac{4}{9}$, \$$\frac{17}{9}$, \$$\frac{1}{9}$? *Ans.* ?

187. CASE II.—When the denominators, or fractional units, are unlike.

1. Add together $\frac{3}{4}$ and $\frac{2}{3}$.

EXPLANATION.—Since the numbers to be added, viz: 3 *fourths* and 2 *thirds*, are unlike, they cannot be added in their present form. Reducing them to a common denominator, or *unit*, by Art. **183**, we obtain $\frac{9}{12}$ and $\frac{8}{12}$, the sum of which, by Case I, is $\frac{17}{12} = 1\frac{5}{12}$. Hence,

OPERATION.

$$\frac{3}{4} = \frac{9}{12}$$
$$\frac{2}{3} = \frac{8}{12}$$
$$\frac{17}{12} = 1\frac{5}{12}.$$

RULE.—*Reduce the fractions to a common denominator, and proceed as in Case I.*

NOTE.—In addition and subtraction, the fractions should be written under each other after the manner of whole numbers.

2. What is the sum of $\frac{4}{5}$ and $\frac{3}{4}$? *Ans.* $\frac{31}{20} = 1\frac{11}{20}$.
3. What is the sum of $\frac{7}{8}$ and $\frac{2}{3}$? *Ans.* $1\frac{13}{24}$.

What is the sum of:

4. $\frac{4}{7}$ and $\frac{3}{5}$? *Ans.* $1\frac{6}{35}$.
5. $\frac{3}{6}$ and $\frac{4}{7}$? *Ans.* $\frac{45}{42}$.
6. $\frac{5}{8}$ and $\frac{6}{9}$? *Ans.* $\frac{93}{72}$.
7. $\frac{3}{11}$ and $\frac{5}{7}$? *Ans.* $\frac{76}{77}$.
8. $\frac{5}{6}$ and $\frac{9}{10}$? *Ans.* $1\frac{11}{15}$.
9. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{6}{5}$? *Ans.* $2\frac{37}{60}$.
10. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{6}{7}$? *Ans.* $2\frac{37}{84}$.
11. $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$? *Ans.* $2\frac{95}{168}$.
12. $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$? *Ans.* $2\frac{313}{504}$.
13. $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{9}{10}$? *Ans.* $2\frac{239}{360}$.

14. Find the sum of $7\frac{2}{3}$ and $9\frac{3}{4}$.

EXPLANATION.—When there are mixed numbers, we add the fractions first, and then add their sum to the sum of the whole numbers. Adding $\frac{9}{12}$ and $\frac{8}{12}$, we get $\frac{17}{12} = 1\frac{5}{12}$; put down the $\frac{5}{12}$ and carry 1 to be added to the sum of the whole numbers, we get $17\frac{5}{12}$.

OPERATION.

$$7\frac{2}{3} = 7\frac{8}{12}$$
$$9\frac{3}{4} = 9\frac{9}{12}$$
$$17\frac{5}{12}.$$

15. Add together $12\frac{1}{2}$ and $15\frac{1}{3}$. *Ans.* $27\frac{5}{6}$.

16. Add together $23\frac{2}{3}$, $18\frac{3}{4}$, and $32\frac{1}{2}$. *Ans.* $74\frac{11}{12}$.

17 A man paid $\$13\frac{1}{2}$ for a pair of pants, $\$17\frac{3}{4}$ for a coat, and $\$5\frac{2}{3}$ for a vest. What did he pay for all?

18. One boy weighs $64\frac{3}{4}$ pounds, another boy $56\frac{3}{5}$, pounds, and the third boy $49\frac{1}{2}$ pounds. What is the total weight of the three boys? *Ans.* $170\frac{17}{20}$ pounds.

19. A man planted $120\frac{3}{8}$ acres in corn, $75\frac{1}{6}$ acres in cotton, $32\frac{1}{8}$ acres in wheat, and $15\frac{5}{12}$ acres in oats. How many acres did he have in cultivation?

Ans. $243\frac{7}{24}$ acres.

Subtraction of Fractions.

Inductive Exercises.

188. Parallelisms.

Whole Numbers.	Fractions.
1. From 4 *fives* take 2 *fives*.	**1.** From 4 *fifths* take 2 *fifths*.
Ans. 2 *fives*.	*Ans.* 2 *fifths*.
2. From 15 *fours* take 8 *fives*.	**2.** From 15 *fourths* take 8 *fifths*.
These are unlike, and must be reduced to like units.	These are unlike, and must be reduced to like units.
15 *fours* = 3 *twenties*. 8 *fives* = 2 *twenties*.	15 *fourths* = 75 *twentieths*, 8 *fifths* = 32 *twentieths*.
Now subtracting like numbers, we get	Now subtracting like numbers, we get
1 *twenty*.	43 *twentieths*.

PRINCIPLE.—*To subtract one number from another, whether whole or fractional, they must be reduced, if not already so, to like units.*

189. CASE I.—When the denominators or fractional units are alike.

1. Subtract $\frac{3}{10}$ from $\frac{9}{10}$.

ANALYSIS.—Since the numbers to be subtracted, viz: 3 *tenths* and 9 *tenths*, have the same unit, 1 *tenth*, we subtract as in whole numbers, and obtain 6 tenths, or $\frac{6}{10} = \frac{3}{5}$, *Ans.*

Hence, the

RULE.—*Take the less numerator from the greater, and place the difference over the common denominator.*

2. What is the difference between $\frac{8}{9}$ and $\frac{3}{9}$?
3. What is the difference between $\frac{7}{12}$ and $\frac{5}{12}$?

How much more is:

4. $\frac{7}{13}$ than $\frac{4}{13}$? *Ans.* $\frac{3}{13}$.
5. $\frac{15}{12}$ than $\frac{7}{12}$? *Ans.* $\frac{2}{3}$.
6. $\frac{13}{15}$ than $\frac{8}{15}$? *Ans.* $\frac{1}{3}$.
7. $\frac{9}{20}$ than $\frac{5}{20}$? *Ans.* ?
8. $\frac{11}{14}$ than $\frac{4}{14}$? *Ans.* ?
9. $\frac{17}{24}$ than $\frac{11}{24}$? *Ans.* ?

190. CASE II.—When the denominators or fractional units are unlike.

1. From $\frac{3}{5}$ take $\frac{1}{2}$.

EXPLANATION.—Since 3 *fifths* and 1 *half* are unlike, they can not be subtracted in their present form. Reducing them to a common denominator, by Art. **183**, we obtain 6 *tenths* and 5 *tenths*, the difference between which is 1 *tenth*, or $\frac{1}{10}$.

OPERATION.

$\frac{3}{5} = \frac{6}{10}$
$\frac{1}{2} = \frac{5}{10}$
$\frac{1}{10}$ *Ans.*

Hence, the

RULE.—*Reduce the fractions to a common denominator, and proceed as in Case I.*

Find the difference between:

2. $\frac{7}{8}$ and $\frac{2}{3}$.	*Ans.* $\frac{5}{24}$.	**9.** $\frac{1}{2}$ and $\frac{11}{24}$.	*Ans.* $\frac{13}{24}$.
3. $\frac{6}{7}$ and $\frac{3}{4}$.	*Ans.* $\frac{3}{28}$.	**10.** $\frac{3}{4}$ and $\frac{3}{13}$.	*Ans.* $\frac{27}{52}$.
4. $\frac{5}{6}$ and $\frac{1}{2}$.	*Ans.* $\frac{1}{3}$.	**11.** $\frac{5}{11}$ and $\frac{2}{9}$.	*Ans.* $\frac{23}{99}$.
5. $\frac{8}{9}$ and $\frac{5}{6}$.	*Ans.* $\frac{1}{18}$.	**12.** $\frac{10}{13}$ and $\frac{3}{8}$.	*Ans.* ?
6. $\frac{9}{11}$ and $\frac{7}{10}$.	*Ans.* $\frac{13}{110}$.	**13.** $\frac{1}{4}$ and $\frac{3}{25}$.	*Ans.* ?
7. $\frac{9}{10}$ and $\frac{7}{7}$.	*Ans.* ?	**14.** $\frac{2}{5}$ and $\frac{10}{43}$.	*Ans.* ?
8. $\frac{3}{4}$ and $\frac{2}{11}$.	*Ans.* ?	**15.** $1\frac{1}{2}$ and $\frac{7}{15}$.	*Ans.* ?

16. From $8\frac{3}{4}$ take $5\frac{2}{3}$.

EXPLANATION.—We first reduce the fractions to a common denominator, then take their difference, and unite it to the difference of the whole numbers. Thus, $\frac{8}{12}$ from $\frac{9}{12}$ leaves $\frac{1}{12}$; 5 from 8 leaves 3; now uniting the 3 and $\frac{1}{12}$, we get $3\frac{1}{12}$.

OPERATION.

$$8\tfrac{3}{4} = 8\tfrac{9}{12}$$
$$5\tfrac{2}{3} = 5\tfrac{8}{12}$$
$$3\tfrac{1}{12}.$$

From:

17. $12\frac{3}{4}$ take $7\frac{2}{5}$.	*Ans.* $5\frac{7}{20}$.	**21.** $45\frac{2}{3}$ take $7\frac{1}{8}$.	*Ans.* $38\frac{13}{24}$.
18. $13\frac{1}{4}$ take $18\frac{2}{6}$.	*Ans.* $5\frac{1}{12}$.	**22.** $164\frac{1}{2}$ take $73\frac{2}{7}$.	*Ans.* ?
19. $35\frac{1}{2}$ take $22\frac{2}{5}$.	*Ans.* $13\frac{1}{10}$.	**23.** $195\frac{3}{4}$ take $126\frac{3}{10}$.	*Ans.* ?
20. $27\frac{2}{3}$ take $23\frac{1}{4}$.	*Ans.* $4\frac{5}{12}$.	**24.** $200\frac{5}{6}$ take $85\frac{7}{20}$.	*Ans.* ?

25. A rope was $487\frac{7}{8}$ feet long, but $17\frac{2}{3}$ feet were cut off; how long was the rope then?

26. From 8 take $\frac{5}{9}$.

EXPLANATION.—8 is equal to 7 and *one*, or, reducing *one* to ninths, 7 and $\frac{9}{9}$. Hence, we write 8 under the form of $7\frac{9}{9}$, and subtract $\frac{5}{9}$ as in the preceding examples.

OPERATION.

$$8 = 7\tfrac{9}{9}$$
$$\tfrac{5}{9} = \tfrac{5}{9}$$
$$7\tfrac{4}{9}.$$

How much more is:

27. 12 than $\frac{7}{9}$?	*Ans.* $11\frac{2}{9}$.	**31.** 64 than $\frac{3}{17}$?	*Ans.* ?
28. 13 than $\frac{5}{11}$?	*Ans.* $12\frac{6}{11}$.	**32.** 75 than $\frac{5}{23}$?	*Ans.* ?
29. 14 than $\frac{3}{8}$?	*Ans.* $13\frac{5}{8}$.	**33.** 84 than $\frac{7}{12}$?	*Ans.* ?
30. 43 than $\frac{7}{13}$?	*Ans.* $42\frac{6}{13}$.	**34.** 125 than $\frac{10}{27}$?	*Ans.* ?

35. From $7\frac{1}{3}$ take $5\frac{3}{4}$.

EXPLANATION.—We reduce the fractions to a common denominator, and as we can not subtract $\frac{9}{12}$ from $\frac{4}{12}$, we take *one*, or $\frac{12}{12}$, from the 7, and add it to $\frac{4}{12}$, making $\frac{16}{12}$. Then we say, $\frac{9}{12}$ from $\frac{16}{12}$ leaves $\frac{7}{12}$, and 5 from 6 (7 less *one*) leaves 1.

OPERATION.

$$7\frac{1}{3} = 7\frac{4}{12}$$
$$5\frac{3}{4} = 5\frac{9}{12}$$
$$1\frac{7}{12}.$$

What is the value of:

36. $8\frac{1}{2} - 3\frac{7}{8}$? *Ans.* $4\frac{5}{8}$.

37. $16\frac{2}{3} - 12\frac{3}{4}$? *Ans.* $3\frac{11}{12}$.

38. $43\frac{1}{3} - 18\frac{3}{5}$? *Ans.* $24\frac{11}{15}$.

39. $68\frac{3}{8} - 49\frac{7}{9}$? *Ans.* $18\frac{43}{72}$.

40. $100\frac{3}{4} - 65\frac{11}{12}$? *Ans.* ?

41. $146\frac{2}{9} - 86\frac{13}{15}$? *Ans.* ?

42. A man had $\$5\frac{1}{4}$ and paid for a knife $\$\frac{4}{5}$; how much did he have left? *Ans.* $\$4\frac{9}{20}$.

43. One melon weighs $25\frac{4}{17}$ pounds, and another weighs $17\frac{4}{15}$ pounds; how much heavier is one than the other? *Ans.* $7\frac{248}{255}$ pounds.

44. Frank and John went fishing; Frank walked $18\frac{11}{320}$ miles and John $2\frac{9}{64}$ miles; how much further did Frank walk than John? *Ans.* $15\frac{143}{160}$ miles.

45. A farmer sold $1\frac{5}{6}$ acres from a field containing $37\frac{3}{8}$ acres; how many acres had he left in the field?

Ans. $35\frac{13}{24}$ acres.

46. One bale of cotton weighs $463\frac{1}{3}$ pounds, and another bale weighs $17\frac{3}{4}$ pounds less; what does the lighter bale weigh? *Ans.* ?

191. In the following examples the whole and one or more parts are given, and the **c** part required. See Art. **89.**

47. The whole is $385\frac{1}{4}$ days, and one part $67\frac{1}{3}$ days; what is the other or **c** part? *Ans.* $317\frac{11}{12}$ days.

48. A man had $\$1673\frac{5}{7}$ and spent $\$356\frac{3}{4}$; how many dollars had he left? *Ans.* $\$1316\frac{27}{28}$.

49. A man had $\$540\frac{1}{2}$ and spent $\$271\frac{3}{4}$ for sheep, and $\$180\frac{2}{3}$ for hogs; how much money did he have left?
Ans. $\$88\frac{1}{12}$.

50. A flag-pole, standing in the water, is 100 feet long; $72\frac{2}{3}$ feet of its length are above the water, and $12\frac{3}{8}$ feet are in the mud below the water; how deep is the water? *Ans.* $14\frac{23}{24}$ ft.

51. Two boys, Charles and Henry, are 400 feet apart; if Charles goes towards Henry $167\frac{5}{6}$ feet, and Henry goes towards Charles $207\frac{3}{4}$ feet, how far apart will they then be? *Ans.* $24\frac{5}{12}$ ft.

Multiplication of Fractions.

Inductive Exercises.

192. Parallelisms.

Whole Numbers.	Fractions.
1. Multiply: 5 *threes* by 4 *threes*.	1. Multiply: 5 *thirds* by 4 *thirds*.
Since $5 \times 4 = 20$, and *three* $\times$ *three* = *nine*, 5 *threes* $\times$ 4 *threes* = 20 *nines*.	Since $5 \times 4 = 20$, and 1 *third* $\times$ 1 *third* = 1 *ninth*, 5 *thirds* $\times$ 4 *thirds* = 20 *ninths*.
2. Multiply: 7 *threes* by 5 *fours*.	2. Multiply: 7 *thirds* by 5 *fourths*.
Since $7 \times 5 = 35$, and *three* $\times$ *four* = *twelve*, 7 *threes* $\times$ 5 *fours* = 35 *twelves*.	Since $7 \times 5 = 35$, and 1 *third* $\times$ 1 *fourth* = 1 *twelfth*, 7 *thirds* $\times$ 5 *fourths* = 35 *twelfths*.

PRINCIPLE.—*To multiply two abstract numbers, whether whole or fractional, we may multiply the numbers regarded as units together for a new unit, and the numerators, or numeral factors, together for a new numerator.*

193. CASE I.—To multiply a fraction by a whole number.

1. Multiply $\frac{5}{6}$ by 3.

EXPLANATION.—The multiplicand is 5 *sixths* and the multiplier 3. 3 times 5 *sixths* are 15 *sixths*, or $\frac{15}{6} = 2\frac{1}{2}$.

Instead of multiplying the factor 5 by 3, we may multiply the unit factor 1 sixth by 3. By Art. **173**, 3 times 1 sixth is 1 half. Hence, 5 sixths $\times 3 = 5$ halves, or $\frac{5}{2} = 2\frac{1}{2}$.

1st. OPERATION.

$\frac{5}{6} \times 3 = \frac{15}{6} = 2\frac{1}{2}$.

2d. OPERATION.

$\frac{5}{6} \times 3 = \frac{5}{2} = 2\frac{1}{2}$.

Hence, the

RULE.—*Multiply the numerator by the whole number, or divide the denominator by the whole number when it can be done without a remainder.*

EXERCISES.

Multiply:

2. $\frac{5}{9}$ by 3.	*Ans.* $1\frac{2}{3}$.	**8.** $\frac{15}{56}$ by 8.	*Ans.* $2\frac{1}{7}$.
3. $\frac{7}{8}$ by 4.	*Ans.* $3\frac{1}{2}$.	**9.** $\frac{24}{63}$ by 7.	*Ans.* $2\frac{2}{3}$.
4. $\frac{3}{11}$ by 6.	*Ans.* $1\frac{7}{11}$.	**10.** $\frac{30}{44}$ by 11.	*Ans.* ?
5. $\frac{4}{7}$ by 7.	*Ans.* 4.	**11.** $\frac{37}{60}$ by 12.	*Ans.* ?
6. $\frac{3}{16}$ by 8.	*Ans.* $1\frac{1}{2}$.	**12.** $\frac{13}{41}$ by 10.	*Ans.* ?
7. $\frac{11}{20}$ by 5.	*Ans.* ?	**13.** $\frac{14}{89}$ by 13.	*Ans.* $4\frac{2}{3}$.

14. If 1 gallon of syrup cost $\frac{5}{8}$ of a dollar, what will 6 gallons cost? *Ans.* $\$3\frac{3}{4}$.

15. If 1 bushel of potatoes cost $\frac{3}{4}$ of a dollar, what will 8 bushels cost? *Ans.* ?

16. If 1 basket holds $\frac{4}{5}$ of a bushel of corn, how many bushels will 8 baskets hold? *Ans.* $6\frac{2}{5}$ bushels.

17. If 1 bucket holds $\frac{6}{7}$ of a gallon of water, how many gallons will 5 buckets hold? *Ans.* ?

18. Multiply $7\frac{3}{4}$ by 5.

We may reduce $7\frac{3}{4}$ to an improper fraction, and multiply as in the preceding exercises. It is generally better, however, to multiply thus: $5 \times \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$. $5 \times 7 = 35$. Now, adding $3\frac{3}{4}$ to 35, we have $38\frac{3}{4}$.

NOTE.—Since $8\frac{1}{3} \times 5 = 5 \times 8\frac{1}{3}$, it is immaterial which we regard as the multiplier.

OPERATION.

$$\begin{array}{r} 7\frac{3}{4} \\ 5 \\ \hline 3\frac{3}{4} \\ 35 \\ \hline 38\frac{3}{4} \end{array}$$

What is the value of:

19. $6\frac{2}{3} \times 7$? *Ans.* $46\frac{2}{3}$.	**24.** $9 \times 18\frac{2}{3}$?	*Ans.* 168.
20. $7\frac{1}{2} \times 3$? *Ans.* $22\frac{1}{2}$.	**25.** $12 \times 13\frac{5}{6}$?	*Ans.* 166.
21. $12\frac{5}{6} \times 9$? *Ans.* $115\frac{1}{2}$.	**26.** $20 \times 11\frac{4}{5}$?	*Ans.* ?
22. $10\frac{3}{4} \times 8$? *Ans.* ?	**27.** $48 \times 30\frac{7}{16}$?	*Ans.* ?
23. $7\frac{3}{5} \times 20$? *Ans.* ?	**28.** $72 \times 123\frac{5}{9}$?	*Ans.* ?

29. What will 32 gallons of brandy cost at $\$1\frac{1}{8}$ per gallon? *Ans.* \$36.

30. What will $15\frac{1}{16}$ barrels of apples cost at \$3 per barrel? *Ans.* $\$45\frac{3}{16}$.

31. What will 556 pounds of cotton amount to at $8\frac{3}{4}$ cents a pound? *Ans.* 4865 cents.

What will be the cost of:

32. 654 pounds of cotton at $7\frac{2}{3}$ c. a pound? *Ans.* 5014 c.
33. 255 pounds of sugar at $9\frac{3}{5}$ c. a pound? *Ans.* 2448 c.
34. 876 yards of prints at $5\frac{3}{4}$ c. a yard? *Ans.* 5037 c.
35. 1260 bushels of corn at $64\frac{2}{3}$ c. a bushel? *Ans.* 81480 c.
36. 570 yards of silk at $\$1\frac{2}{5}$ a yard? *Ans.* \$798.

194. CASE II.—To multiply a fraction by a fraction.

1. Multiply $\frac{5}{7}$ by $\frac{2}{3}$.

EXPLANATION.—Multiplication means taking one number as many times as there are units in another. In $\frac{2}{3}$ there are only $\frac{2}{3}$ of a unit. Hence, we are required to take $\frac{5}{7}$ $\frac{2}{3}$ of 1 *time*. Now, 1 time $\frac{5}{7}$ is $\frac{5}{7}$, hence $\frac{2}{3}$ of 1 time $\frac{5}{7}$ is $\frac{2}{3}$ of $\frac{5}{7} = \frac{10}{21}$.

OPERATION.

$\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21}$.

Hence, the

RULE.—*Multiply the numerators together for a numerator, and the denominators together for a denominator.*

Or, *Regard* $\times$ *as meaning* ***of,*** *and proceed as in the reduction of compound fractions to simple ones.*

NOTE.—Mixed numbers must be reduced to improper fractions.

Multiply:

2.	$\frac{4}{5}$ by $\frac{2}{3}$.	*Ans.* $\frac{8}{15}$.	**7.**	$7\frac{1}{4}$ by $8\frac{1}{2}$.	*Ans.*	$61\frac{5}{8}$.
3.	$\frac{1}{4}$ by $\frac{7}{9}$.	*Ans.* $\frac{7}{36}$.	**8.**	$7\frac{1}{2}$ by $5\frac{3}{5}$.	*Ans.*	42.
4.	$\frac{3}{8}$ by $\frac{6}{9}$.	*Ans.* $\frac{1}{4}$.	**9.**	$6\frac{3}{8}$ by $3\frac{1}{3}$.	*Ans.*	$21\frac{1}{4}$.
5.	$\frac{5}{7}$ by $\frac{4}{15}$.	*Ans.* $\frac{4}{21}$.	**10.**	$7\frac{3}{8}$ by $2\frac{3}{4}$.	*Ans.*	$20\frac{9}{32}$.
6.	$4\frac{1}{2}$ by $\frac{1}{8}$.	*Ans.* $\frac{9}{16}$.	**11.**	$4\frac{1}{3}$ by $2\frac{1}{4}$.	*Ans.*	?

What will be the cost of:

12. 12 yards of cloth at 8 c. a yard? *Ans.* 96 c.

13. 4 barrels of cider at $\$3\frac{1}{4}$ a barrel? *Ans.* \$13.

14. $25\frac{2}{3}$ pounds of coffee at 12 c. a pound? *Ans.* 308 c.

15. $1\frac{7}{8}$ barrels of flour at $\$5\frac{3}{4}$ a barrel? *Ans.* $\$10\frac{25}{32}$.

16. $9\frac{1}{8}$ bushels of corn at $62\frac{1}{2}$ c. a bushel? *Ans.* $570\frac{5}{16}$ c.

17. $240\frac{3}{11}$ acres of land at $\$25\frac{5}{8}$ an acre? *Ans.* $\$6156\frac{87}{88}$.

What is the cost of:

18. 16 pounds of cheese at $8\frac{1}{2}$ c. a pound? *Ans.* 136 c.

19. $15\frac{2}{3}$ yards of cambric at 15 c. per yard? *Ans.* 235 c.

20. $11\frac{1}{9}$ cords of wood at $\$3\frac{1}{2}$ per cord? *Ans.* $\$38\frac{8}{9}$.

21. $15\frac{1}{2}$ yards of broadcloth at $\$3\frac{11}{15}$ a yard? *Ans.* $\$57\frac{13}{15}$.

22. $15\frac{3}{4}$ yards of ribbon at 40 c. per yard? *Ans.* 630 c.

23. $8\frac{3}{4}$ yards of silk at $\$\frac{5}{10}$ per yard? *Ans.* $\$4\frac{3}{8}$.

24. 348 pounds of cotton at $7\frac{2}{8}$ c. per pound? *Ans.* ?

Division of Fractions.

Inductive Exercises.

195. Parallelisms.

Whole Numbers.	Fractions.
1. Divide 15 *fours* by 3 *fours*.	1. Divide 15 *fourths* by 3 *fourths*.
Ans. $15 \div 3 = 5$.	*Ans.* $15 \div 3 = 5$.
2. Divide 15 *fours* by 8 *fives*.	2. Divide 15 *fourths* by 8 *fifths*.
Reducing to like units:	Reducing to like units:
15 *fours* = 3 *twenties*,	15 *fourths* = 75 *twentieths*,
8 *fives* = 2 *twenties*.	8 *fifths* = 32 *twentieths*,
Now,	Now,
3 *twenties* ÷ 2 *twenties*	75 *twentieths* ÷ 32 *twentieths*
$= 3 \div 2 = 1\frac{1}{2}$.	$= 75 \div 32 = 2\frac{11}{32}$.

Principle.—*To find how often one number is contained in another, whether whole or fractional, they must be reduced, if not already so, to like units.*

196. Case I.—To divide a fraction by a whole number.

1. Divide $\frac{20}{3}$ by 5.

Explanation.—The dividend is 20 *thirds*, the divisor 5. Now, 20 *thirds* ÷ 5 = 4 *thirds* = $\frac{4}{3} = 1\frac{1}{3}$.

Instead of dividing the factor 20 by 5, we may divide the unit 1 *third* by 5. By Art. **173**, 1 *third* ÷ 5 = 1 *fifteenth*. Hence, 20 *thirds* ÷ 5 = 20 *fifteenths* = $\frac{20}{15} = \frac{4}{3} = 1\frac{1}{3}$.

1st operation.

$\frac{20}{3} \div 5 = \frac{4}{3} = 1\frac{1}{3}$.

2d operation.

$\frac{20}{3} \div 5 = \frac{20}{15} = \frac{4}{3} = 1\frac{1}{3}$.

RULE.—*Divide the numerator or multiply the denominator by the whole number.*

EXERCISES.

Divide:

2. $\frac{2}{3}$ by 2. *Ans.* $\frac{1}{3}$.
3. $\frac{3}{4}$ by 2. *Ans.* $\frac{3}{8}$.
4. $\frac{8}{9}$ by 6. *Ans.* $\frac{4}{27}$.
5. 12 *fifths* by 4. *Ans.* $\frac{3}{5}$.
6. 15 *halves* by 5. *Ans.* $1\frac{1}{2}$.
7. $\frac{63}{16}$ by 9. *Ans.* $\frac{7}{16}$.
8. $\frac{18}{19}$ by 6. *Ans.* ?
9. $\frac{61}{3}$ by 8. *Ans.* $2\frac{2}{3}$.
10. $\frac{3}{7}$ by 9. *Ans.* $\frac{1}{21}$.
11. $\frac{10}{13}$ by 12. *Ans.* ?
12. $\frac{501}{71}$ by 25. *Ans.* ?
13. 7 *ninths* by 4. *Ans.* $\frac{7}{36}$.

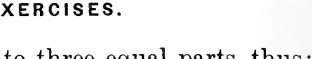
14. If 4 tops cost $\$\frac{3}{5}$, what will 1 top cost? *Ans.* $\$\frac{3}{20}$.
15. If 3 melons cost $\$\frac{6}{7}$, what will 1 melon cost? *Ans.* $\$\frac{2}{7}$.
16. If 5 spellers cost $\$\frac{10}{11}$, what will 1 speller cost? *Ans.* $\$\frac{2}{11}$.

CASE II.—To divide 1 by a fraction.

INDUCTIVE EXERCISES.

197. Divide a bar of soap into three equal parts, thus:

How often are 2 *parts* contained in 3 *parts?* *Ans.* $\frac{3}{2}$.



What stands for 2 parts?
What stands for 3 parts? *Ans.* 1.
How often, then, is $\frac{2}{3}$ contained in 1? *Ans.* $\frac{3}{2}$.
How often is $\frac{3}{4}$ contained in 1? *Ans.* $\frac{4}{3}$.
Why? *Ans.* Because $\frac{3}{4} = 3$ *fourths*, and $1 = 4$ *fourths*, and 4 *fourths* $\div$ 3 *fourths* $= \frac{4}{3}$.
How often is $\frac{5}{7}$ contained in 1? *Ans.* $\frac{7}{5}$ *times*, because 5 *sevenths* is contained in 7 *sevenths* $\frac{7}{5}$ *times.*

198. The Reciprocal of a Fraction is the result of interchanging the places of its terms. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{3}{4}$, $\frac{4}{3}$; of $\frac{5}{7}$, $\frac{7}{5}$; of 4, $\frac{1}{4}$; of $2\frac{1}{3}$ or $\frac{7}{3}$, $\frac{3}{7}$; etc.

Inverting a fraction is taking its *reciprocal.*

199. From the preceding articles we derive the

RULE.—*To find how often a fraction is contained in one, or* 1, *we take its reciprocal, or invert it.*

EXERCISES.

How often is:

1. $\frac{3}{8}$ contained in 1? *Ans.* $\frac{8}{3}$ times.
2. $\frac{11}{15}$ contained in 1? *Ans.* $\frac{15}{11}$ times.
3. $\frac{9}{5}$ contained in 1? *Ans.* $\frac{5}{9}$ times.
4. $\frac{4}{8}$ contained in 1? *Ans.* 2 times.
5. Is $\frac{4}{8}$ equal to $\frac{1}{2}$? Why?

   Ans. Because each is contained in 1 2 times.
6. Is $\frac{3}{12}$ equal to $\frac{5}{20}$? Why?
7. Is $\frac{7}{21}$ equal to $\frac{8}{24}$? Why?
8. Is $\frac{7}{35}$ equal to $\frac{13}{65}$? Why?

How many:

9. $\frac{2}{3}$ will it take to make 1? *Ans.* 4.
10. $\frac{3}{15}$ will it take to make 1? *Ans.* 5.
11. $\frac{4}{17}$ will it take to make 1? *Ans.* $4\frac{1}{4}$.

200. CASE III.—To divide a whole number or a fraction by a fraction.

1. Divide 5 by $\frac{2}{3}$.

EXPLANATION.—By Case II. $\frac{2}{3}$ is contained in 1 $\frac{3}{2}$ times; hence, in 5 it is contained 5 times $\frac{3}{2}$ or $\frac{15}{2} = 7\frac{1}{2}$.

OPERATION.

$$\frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$$

2. Divide $\frac{3}{4}$ by $\frac{5}{7}$.

EXPLANATION.—By Case II, $\frac{5}{7}$ is contained in 1 $\frac{7}{5}$ times; hence, in $\frac{3}{4}$ it is contained $\frac{3}{4}$ times $\frac{7}{5}$ or $\frac{21}{20} = 1\frac{1}{20}$.

OPERATION.

$$\frac{3}{4} \times \frac{7}{5} = \frac{21}{20} = 1\frac{1}{20}$$

Hence, the

RULE.—*Invert the divisor and proceed as in multiplication of fractions.*

NOTE.—This rule is also applicable to Case 1.

EXERCISES.

Divide:

3. 18 by $\frac{2}{3}$.	*Ans.* 27.	**10.** $1\frac{1}{2}$ by $\frac{1}{2}$.	*Ans.* 3.
4. 20 by $\frac{1}{4}$.	*Ans.* 80.	**11.** $3\frac{1}{3}$ by $\frac{2}{3}$.	*Ans.* 5.
5. 6 by $\frac{3}{5}$.	*Ans.* 10.	**12.** $5\frac{1}{4}$ by $\frac{3}{4}$.	*Ans.* 7.
6. 15 by $\frac{5}{8}$.	*Ans.* 24.	**13.** $2\frac{1}{2}$ by $\frac{5}{6}$.	*Ans.* 3.
7. $\frac{8}{9}$ by $\frac{3}{4}$.	*Ans.* $\frac{32}{27}$.	**14.** $7\frac{1}{2}$ by $1\frac{1}{4}$.	*Ans.* ?
8. $\frac{5}{8}$ by $\frac{3}{4}$.	*Ans.* $\frac{5}{6}$.	**15.** $5\frac{1}{4}$ by $3\frac{1}{2}$.	*Ans.* $1\frac{1}{2}$.
9. $\frac{3}{7}$ by $\frac{2}{3}$.	*Ans.* $\frac{9}{14}$.	**16.** $6\frac{1}{2}$ by $1\frac{2}{3}$.	*Ans.* $3\frac{9}{10}$.

17. Divide $13\frac{1}{2}$ by $7\frac{1}{3}$. *Ans.* $1\frac{37}{44}$.
18. Divide $24\frac{1}{5}$ by $2\frac{3}{4}$. *Ans.* $8\frac{4}{5}$.
19. Divide $15\frac{3}{7}$ by $3\frac{3}{5}$. *Ans.* ?
20. How many $\frac{3}{4}$ make $7\frac{1}{2}$? *Ans.* 10.
21. How many $1\frac{2}{9}$ make $14\frac{2}{3}$? *Ans.* 12.
22. How many $\frac{7}{8}$ make $10\frac{1}{2}$? *Ans.* ?
23. How many $\frac{2}{5}$ make $\frac{3}{4}$? *Ans.* $1\frac{7}{8}$.

What will 1 yard of cloth cost:

24. If 4 yards cost 12 cents? *Ans.* ?
25. If 4 yards cost $12\frac{1}{2}$ cents? *Ans.* $3\frac{1}{8}$c.
26. If $4\frac{1}{2}$ yards cost 12 cents? *Ans.* $2\frac{2}{3}$c.
27. If $4\frac{1}{2}$ yards cost $12\frac{1}{2}$ cents? *Ans.* $2\frac{7}{9}$c.
28. If $3\frac{1}{3}$ yards cost $13\frac{1}{3}$ cents? *Ans.* 4c.
29. If $5\frac{1}{4}$ yards cost $31\frac{1}{2}$ cents? *Ans.* ?
30. If $3\frac{1}{8}$ yards cost $\frac{5}{6}$? *Ans.* $\frac{4}{15}$.
31. If $5\frac{1}{3}$ yards cost $\frac{4}{7}$? *Ans.* ?
32. If $3\frac{1}{7}$ yards cost $2\frac{3}{4}$? *Ans.* ?
33. If $5\frac{1}{3}$ yards cost $3\frac{1}{8}$? *Ans.* ?

Written Exercises.

201. *An important class of problems involving Multiplication and Division of Fractions.* (See Art. **142.**) *

1. If $1\frac{2}{3}$ yards of cloth cost $\$1\frac{3}{4}$, what will $2\frac{1}{5}$ yards cost?

Explanation. — Since $1\frac{2}{3}$ yards cost $\$1\frac{3}{4}$, we divide $1\frac{3}{4}$ by $1\frac{2}{3}$ to get the cost of 1 yard, which gives $\$\frac{21}{20}$. Now since 1 yard costs $\$\frac{21}{20}$, we multiply $\frac{21}{20}$ by $2\frac{1}{5}$ to get the cost of $2\frac{1}{5}$ yards, and obtain $\$2\frac{31}{100}$, *Ans.*

Operation.

$1\frac{3}{4} \div 1\frac{2}{3} = \frac{21}{20}$.

$\frac{21}{20} \times 2\frac{1}{5} = \frac{231}{100} = 2\frac{31}{100}$.

2. If $2\frac{1}{2}$ yards of cloth cost $\$3\frac{3}{4}$, what will $1\frac{2}{3}$ yards cost? *Ans.* $\$2\frac{1}{2}$.

3. If $\frac{3}{4}$ yard of cloth cost 21 cents, what will $2\frac{1}{7}$ yards cost? *Ans.* 60 cents.

4. If $2\frac{1}{5}$ yards of cloth cost $\$1\frac{3}{8}$, what will $2\frac{1}{2}$ yards cost? *Ans.* $\$1\frac{9}{16}$.

5. If $3\frac{1}{3}$ yards of cloth cost $\$4\frac{4}{9}$, what will $2\frac{1}{4}$ yards cost? *Ans.* $3.

6. If $\frac{2}{3}$ yard of cloth cost $3\frac{1}{2}$ cents, what will $13\frac{1}{5}$ yards cost? *Ans.* ?

How much will:

7. 12 yards of cloth cost if $3\frac{1}{2}$ yards cost 21 cents? *Ans.* 72 cents.

8. $15\frac{1}{2}$ bushels of corn cost if $3\frac{1}{3}$ bushels cost $\$1\frac{2}{5}$? *Ans.* $6.51.

9. $7\frac{1}{3}$ gallons of syrup cost if $2\frac{1}{4}$ gallons cost $\$1\frac{4}{5}$? *Ans.* $\$5\frac{13}{15}$.

10. $13\frac{1}{5}$ pounds of beef cost, if $\frac{3}{10}$ pounds cost $1\frac{1}{2}$ c.?

*Before the pupils begin these exercises, let them solve six or eight of the examples under Art. **142.**

11. How much will $25\frac{1}{3}$ acres of land cost, if $2\frac{1}{6}$ acres cost \$26? *Ans.* \$304.

12. How much will $10\frac{3}{4}$ barrels of flour cost, if $1\frac{1}{12}$ barrels cost $\$8\frac{2}{3}$? *Ans.* \$86.

13. A farmer bought $6\frac{1}{4}$ pounds of nails for 25 c., and desires to get $8\frac{1}{2}$ pounds more at the same rate; how much will they cost? *Ans.* 32 c.

14. A farmer's price for $250\frac{1}{2}$ acres of land is \$2505; what is his price for $175\frac{3}{4}$ acres? *Ans.* $\$1757\frac{1}{2}$.

202. Parallel Problems.

1.[m] Reduce to lowest terms $\frac{6}{18}$, $\frac{12}{24}$, $\frac{27}{36}$.

2. Reduce to lowest terms $\frac{105}{189}$, $\frac{672}{1008}$. *Ans.* $\frac{5}{9}$, $\frac{2}{3}$.

3.[m] Change $\frac{2}{3}$ to *twelfths*; $\frac{3}{4}$ to *twentieths*.

4. Change $\frac{17}{24}$ to *one hundred sixty-eighths*. *Ans.* $\frac{119}{168}$.

5.[m] What is the number whose *seventh* is 12?

6. What is the number whose *thirty-sixth* is 95? *Ans.* 3420.

7.[m] Six-sevenths of a number is 12; what is the number?

8. $\frac{23}{47}$ of a number is 138; what is the number? *Ans.* 282.

9.[m] What is the weight of a beef, if $\frac{3}{4}$ of it weighs 600 pounds?

10. What is the weight of a beef, if $\frac{37}{41}$ of it weighs 629 pounds? *Ans.* 697 pounds.

11.[m] If $\frac{4}{5}$ of an acre of land cost \$12, what will one acre cost?

12. If $\frac{43}{50}$ of a lot is valued at \$3225, what is the price of the whole lot? *Ans.* \$3750.

13.[m] If 3 *fourths* of a pound of sugar cost 9 cents, what will 1 *fourth* of a pound cost?

14. If $\frac{2}{3}\frac{5}{7}$ of a load of cotton cost \$175, what would $\frac{1}{3}\frac{}{7}$ of the load cost? *Ans.* \$7.

15. Reduce $\frac{5}{1}\frac{0}{6}\frac{8}{}$ to a mixed number. *Ans.* $31\frac{3}{4}$.

16.[m] What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? $\frac{1}{3}$ and $\frac{1}{4}$?

17. What is the sum of $\frac{6}{7}$ and $1\frac{2}{3}$? *Ans.* $1\frac{7}{9}\frac{1}{}$.

What is the number whose **c** parts are:

18.[m] 5 and $\frac{3}{4}$? $7\frac{1}{2}$ and $\frac{1}{4}$? $9\frac{2}{5}$ and $3\frac{1}{1}\frac{}{0}$? $6\frac{1}{2}$ and $5\frac{5}{8}$?

19. $18\frac{3}{4}$ and $36\frac{5}{1}\frac{}{2}$? $481\frac{2}{3}$ and $1967\frac{7}{8}$? *Ans.* $55\frac{1}{6}$, $678\frac{1}{2}\frac{3}{4}$.

20. A merchant has two pieces of prints, one contains $23\frac{3}{4}$ yards and the other $48\frac{4}{5}$ yards; how many yards in both pieces? *Ans.* $72\frac{1}{2}\frac{1}{0}$ yards.

21.[m] What is the difference between $\frac{1}{2}$ and $\frac{1}{4}$? 7 and $\frac{2}{3}$?

22. What is the difference between $\frac{1}{1}\frac{3}{5}$ and $\frac{1}{2}\frac{7}{0}$? *Ans.* $\frac{3}{6}\frac{1}{0}$.

If one of the parts of:

23.[m] 8 is $1\frac{1}{3}$, what is the **c** part?

24. 12 is $7\frac{3}{8}$, what is the **c** part? *Ans.* $4\frac{5}{8}$.

25.[m] \$8$\frac{1}{3}$ is \$5, what is the **c** part?

26. A man had \$$125\frac{3}{4}$ and spent \$$83\frac{1}{3}$; how much had he left? *Ans.* \$$42\frac{5}{1}\frac{}{2}$.

27. From $134\frac{1}{2}$ gallons of water there were drawn off $117\frac{6}{7}$ gallons; how many gallons were left? *Ans.* $16\frac{9}{1}\frac{}{4}$.

28.[m] If two of the parts of 8 are 2 and $3\frac{1}{2}$, what is the **c** part?

29. If two of the parts of $165\frac{3}{4}$ are $17\frac{1}{3}$ and $88\frac{5}{6}$, what is the **c** part? *Ans.* $59\frac{7}{1}\frac{}{2}$.

30. Three boys together weigh $210\frac{1}{2}$ pounds. The first boy weighs $71\frac{2}{3}$ pounds, and the second $69\frac{3}{4}$ pounds; what does the third boy weigh? *Ans.* $69\frac{1}{1}\frac{}{2}$ pounds.

31.[m] What will be the cost of 9 apples at 2 c. apiece? Of 12 peaches at $\frac{2}{3}$ c. apiece? Of 8 gallons of syrup at \$$\frac{3}{4}$ a gallon? Of 9 bushels of corn at \$$\frac{5}{6}$ a bushel?

32. What will 568 pounds of cotton cost at $9\frac{3}{4}$ c. per pound? *Ans.* 5538 c.

33. What will $18\frac{3}{4}$ pounds of butter cost at $18\frac{3}{4}$ c. per pound? *Ans.* $351\frac{9}{16}$ c.

34. What is the number whose **c** factors are $5\frac{1}{4}$ and $5\frac{1}{7}$? *Ans.* 27.

35.[m] What will be the cost of 1 yard of cloth if 4 yards cost $\$1\frac{2}{3}$? If 4 yards cost $\$\frac{3}{5}$?

36. What will be the cost of 1 yard of cloth if $3\frac{1}{2}$ yards cost $25\frac{2}{3}$ c.? *Ans.* $7\frac{1}{3}$ c.

37.[m] If a man travels 4 miles in 1 hour, how far will he go in $3\frac{2}{3}$ hours?

38. If a horse travels $6\frac{1}{3}$ miles in 1 hour, how far will he go in $5\frac{1}{4}$ hours? *Ans.* $33\frac{1}{4}$ miles.

203. Questions for Review.

What is: 1. A fractional unit? 2. A fraction? 3. The terms? 4. The denominator? 5. The numerator? 6. A proper fraction? 7. An improper fraction? 8. A mixed number? 9. The value of a fraction? 10. A compound fraction? 11. The reciprocal of a fraction?

How may a fractional unit be: 1. Multiplied by a whole number? 2. Divided by a whole number?

Why is the value of a fraction not changed by: 1. Multiplying both terms by the same number? 2. Dividing both terms by the same number?

What is reduction of fractions? Give the rule for reducing: 1. A fraction to its lowest terms. 2. A whole number to a fraction. 3. A fraction to higher terms. 4. A mixed number to an improper fraction. 5. An improper fraction to a whole or mixed number. 6. Compound fractions to simple ones. 7. Fractions to a common denominator. 8. Fractions to their least common denominator.

What is the principle of: 1. Addition? 2. Subtraction? 3. Multiplication? 4. Division?

What is the rule for: 1. Addition? 2. Subtraction? 3. Multiplication? 4. Division?

How do we find how often a fraction is contained in 1? In division of fractions, why do we invert the divisor?

Decimal Fractions.

Inductive Exercises.

204. If an orange, an apple, a number, or a bar of soap be divided into ten equal parts, what is one of the parts called? *Ans.* 1 *tenth.* What are two of the parts called? Three of the parts? Four? Five?

If, now, each of these *tenths* be divided into *ten* equal parts, what is one of the parts called? *Ans.* 1 *hundredth.* What are two of the parts called? Three of the parts? Four? Five?

If, now, each of these hundredths be divided into ten equal parts, what is one of the parts called?

Ans. 1 *thousandth.*

In the number 327, what is the unit of 7? *Ans. one.* See Art. **31.** What is the unit of 2? Of 3? Is the unit of 2 *one-tenth* of the unit of 3? Is the unit of 7 *one-tenth* of the unit of 2?

If, now, we write other figures after 7, thus: 327568, will the unit of 5 be 1 *tenth* of the unit of 7? Will the unit of 6 be 1 *tenth* of the unit of 5? Will the unit of 8 be 1 *tenth* of the unit of 6?

Placing a point (.) after 7, thus: 327.568, indicates that its unit is *one*; what, then, is the unit of 5? Of 6? Of 8?

1. What, then, is denoted by 327.568?

Ans. 327 *and* 5 tenths 6 hundredths 8 thousandths.

Or, 327 *and* $\frac{5}{10} + \frac{6}{100} + \frac{8}{1000} = 327$ and $\frac{568}{1000}$, or 327 *and* 568 thousandths.

2. What is denoted by 125.34?

Ans. 125 *and* 34 hundredths.

3 What is denoted by 804.05?

Ans. 804 *and* 5 hundredths.

4. What is denoted by .125? *Ans.* 125 thousandths.

Definitions.

205. A Decimal Fraction is a fraction whose denominator is always 10, 100, 1000, etc., or 1 with 0's annexed.

Decimal Fractions are usually called *decimals*.

206. A Decimal is usually expressed by writing only the *numerator* with a period and a certain number of 0's before it; the denominator being *understood*, but not written.

Thus:

1.	$\frac{1}{10}$	is written	.1	7.	$\frac{7}{1000}$	is written	.007
2.	$\frac{2}{10}$	"	.2	8.	$\frac{27}{1000}$	"	.027
3.	$\frac{7}{10}$	"	.7	9.	$\frac{134}{1000}$	"	.134
4.	$\frac{1}{100}$	"	.01	10.	$\frac{5}{10000}$	"	.0005
5	$\frac{3}{100}$	"	.03	11.	$\frac{304}{10000}$	"	.0304
6.	$\frac{19}{100}$	"	.19	12.	$\frac{25}{1000000}$	"	.000025

The **period** placed before decimals is called the *decimal point;* it is also called *separatrix*, as it *separates* the decimal from the whole number.

207. A Mixed Number is a number formed of a whole number and a decimal, as 4.25, of which 4 is the whole number, and .25, or 25 hundredths, is the decimal.

Principles.

208. 1°. Decimals decrease from left to right in a tenfold ratio, just as whole numbers do.

Hence, the value of a decimal figure depends upon its distance from the decimal point. Thus, .07 is $\frac{7}{100}$; .007 is $\frac{7}{1000}$.

2°. The *first* place on the right of the decimal point is that of *tenths;* the *second* place *hundredths;* the *third, thousandths;* the *fourth, ten-thousandths;* the *fifth, hundred-thousandths;* the *sixth, millionths*, etc.

Decimal Numeration Table.

Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.
5	4	3	2	1	.	1	2	3	4

3°. The denominator (understood) of a decimal is 1 with as many 0's annexed as there are figures or places in the decimal.

What is the denominator of:

1. .6? *Ans.* 10.	4. .027?	7. .0325?
2. .43? *Ans.* 100.	5. .009?	8. .53207?
3. .07? *Ans.* 100.	6. .0005?	9. .00506?

209. Expressing decimals as common fractions.

1. Express .0205 as a common fraction.

Writing the denominator, we have $\frac{0205}{10000}$. Now, removing the *point* and the *cipher* following it, we obtain $\frac{205}{10000}$, *Ans.*

Express as a common fraction:

2. .0027.	*Ans.* $\frac{27}{10000}$.	5. .00325.	*Ans.* $\frac{325}{100000}$.
3. 3.005.	*Ans.* $3\frac{5}{1000}$.	6. 54.0107.	*Ans.* ?
5. 75.04.	*Ans.* $75\frac{4}{100}$.	7. 84.605.	*Ans.* ?

210. Exercises in Numeration and Notation.

1. Read .025. *Ans.* 25 thousandths.
2. Read 13.000325. *Ans.* 13 *and* 325 millionths.

Hence, the

Rule.—*Decimals are read as they would be if expressed as common fractions.*

Note.—In case of mixed numbers, the decimal point is called *and*, which is *emphasized* and *dwelt upon*, to indicate the termination of the whole number, and the beginning of the decimal.

Read:

3. .0605. *Ans.* 605 ten-thousandths.
4. 24.12. *Ans.* 24 *and* 12 hundredths.
5. 3.017. *Ans.* 3 *and* 17 thousandths.
6. 125.01. *Ans.* 125 *and* 1 hundredth.
7. .004. *Ans.* 4 thousandths.
8. 25.036.
9. 20.03.
10. 101.101.
11. 73.00576.
12. 184.37504.

Write the following in the decimal notation:

13. 8 tenths.
14. 24 hundredths.
15. 7 hundredths.
16. 5 *and* 16 thousandths.
17. 12 *and* 3 hundredths.
18. 3 *and* 2 tenths.
19. 15 *and* 2 hundredths.
20. 7 thousandths.
21. 325 ten-thousandths.
22. 4075 millionths.

Principles.

211. Since $\frac{7}{10} = \frac{70}{100} = \frac{700}{1000} = \frac{7000}{10000}$, etc.

We have $.7 = .70 = .700 = .7000$, etc.

Hence,

1°. *Annexing one or more ciphers to a decimal does not alter its value.*

212. *Ten* times 25.78 is 257.8.

For 10 *times* 2 tens 5 ones 7 tenths 8 hundredths are 2 hunds. 5 tens 7 ones 8 tenths, or 257.8. Hence,

2°. *To multiply a decimal by* 10, 100, 1000, *etc., we remove the decimal point one, two, three, etc., places to the right.*

213. Reversing the foregoing principle, we have

3°. *To divide a decimal by* 10, 100, 1000, *etc., we remove the decimal point one, two, three, etc., places to the left.*

Exercises.

214. Which is the more:

1. .27 or .2700?
2. 4.6 or 4.60?
3. 43.063 or 43.06300?
4. 705.47 or 705.47000?

Multiply:

5. 4.35 by 10. *Ans.* 43.5.
6. 75.07 by 1000. *Ans.* 75070.
7. .0374 by 100. *Ans.* 3.74.
8. .00305 by 1000. *Ans.*?
9. 350.47 by 1000. *Ans.*?
10. 47.319 by 10000. *Ans.*?

Divide:

11. 3.5 by 100. *Ans.* .035.
12. 64.5 by 10. *Ans.* 6.45.
13. 13 by 1000. *Ans.* .013.
14. 47.365 by 100. *Ans.*?
15. .002576 by 1000. *Ans.*?
16. 32547.1 by 100000. *Ans.*?

215. To reduce a decimal to a common fraction in its lowest terms.

1 Reduce .125 to a common fraction in its lowest terms.

OPERATION.—$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$.

Hence, the

RULE.—*Express the decimal as a common fraction, and reduce it to its lowest terms.*

Reduce the following to common fractions in their lowest terms:

2. .75.	*Ans.* $\frac{3}{4}$.	**7.** 63.6.	*Ans.* ?
3. 8.25.	*Ans.* $8\frac{1}{4}$.	**8.** 63.600.	*Ans.* ?
4. .035.	*Ans.* $\frac{7}{200}$.	**9.** 47.3125.	*Ans.* ?
5. .0625.	*Ans.* $\frac{1}{16}$.	**10.** .001875.	*Ans.* ?
6. 34.375.	*Ans.* $34\frac{3}{8}$.	**11.** 11.3125.	*Ans.* ?

216. To reduce common fractions to decimals.

1. Reduce $\frac{3}{8}$ to a decimal.

EXPLANATION. — 3 = 30 tenths; 30 tenths ÷ 8 = 3 tenths and 6 tenths over. 6 tenths = 60 hundredths; 60 hundredths ÷ 8 = 7 hundredths and 4 hundredths over. 4 hundredths = 40 thousandths, and this ÷ 8 = 5 thousandths. Hence, the answer is 3 tenths, 7 hundredths, 5 thousandths = .375.

OPERATION.

8)3000

.375

Hence, the

RULE.—*Annex ciphers to the numerator, divide by the denominator, and from the right of the quotient point off as many decimal figures as there are ciphers annexed.*

Reduce to decimals:

2. $\frac{3}{4}$.	*Ans.* .75.	**7.** $9\frac{3}{5}$.	*Ans.* 9.6.
3. $\frac{5}{8}$.	*Ans.* .625.	**8.** $\frac{5}{160}$.	*Ans.* .03125.
4. $\frac{3}{7}$.	*Ans.* .428+.	**9.** $13\frac{7}{16}$.	*Ans.* ?
5. $5\frac{1}{2}$.	*Ans.* 5.5.	**10.** $\frac{17}{200}$.	*Ans.* ?
6. $7\frac{1}{4}$.	*Ans.* 7.25.	**11.** $\frac{13}{32}$.	*Ans.* ?

Addition of Decimals.

217. Since decimals increase and decrease regularly by the scale of ten, they are evidently added like whole numbers. Hence, the

Rule.—*Write the numbers so that points shall stand under points, tenths under tenths, hundredths under hundredths, etc., and add as in whole numbers.*

1. Add together 5.75, 16.263, 143.098, and .96.

OPERATION.

5.75
16.263
143.098
.96
166.071 *Ans.*

Add together:

2. 53.246, 44.82, 706.4, 49.82, and .5. *Ans.* 854.786.

3. 58.07, 43.9, .84, .679, and 9.3. *Ans.* 112.789.

4. 9.74, 16.07, 924., 75.24, and 879. *Ans.* 1025.929.

5. 170., 309.6, 58.754, 3.7, and .0349. *Ans.* 542.0889.

6. 23 *and* 7 hund'ths, 5 *and* 9 tenths, 271 *and* 46 thous'ths, and 133 *and* 575 ten-thous'ths. *Ans.* 433.0735.

7. 27 hund'ths, 83 thous'ths, 984 thous'ths, 7 *and* 8 hund'ths, and 74 *and* 125 ten-thous'ths. *Ans.* 82.4295.

8. 43 *and* 9 tenths, 13 *and* 13 thous'ths, 61 hund'ths, 5 *and* 17 thous'ths, and 425 *and* 78 hund'ths. *Ans.* 488.320.

Note.—In the following, first reduce the fractions to decimals by Art. **216**, and then add.

Find the sum of:

9. $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$. *Ans.* 1.875.

10. $3\frac{1}{2}$, $9\frac{3}{5}$, $7\frac{1}{4}$, and $16\frac{1}{5}$. *Ans.* 36.55.

11. $\frac{7}{8}$, $5\frac{2}{5}$, $16\frac{7}{20}$, and $\frac{11}{32}$. *Ans.* 22.96875.

12. $253\frac{1}{2}$, $187\frac{3}{4}$, $95\frac{2}{5}$, and $3\frac{3}{8}$. *Ans.* 540.025.

Subtraction of Decimals.

218. Decimals are subtracted like whole numbers for the same reason that they are added as such.

Hence, the

Rule.—*Write the subtrahend under the minuend, so that points shall stand under points, tenths under tenths, hundredths under hundredths, etc., and subtract as in whole numbers.*

1. From 34.046 take 9.78.

OPERATION.

$$\begin{array}{r} 34.046 \\ 9.780 \\ \hline 24.266 \end{array}$$ *Ans.*

2. From 41. take 23.35.

OPERATION.

$$\begin{array}{r} 41.00 \\ 23.35 \\ \hline 17.65 \end{array}$$

We write ciphers above the 3 and 5, as there are *no* tenths and *no* hund'ths in the minuend.

From:

3. 34.16 take 17.75. *Ans.* 16.41.
4. 9.6 take 7.035. *Ans.* 2.565.
5. .7 take .368. *Ans.* .332.
6. 87.946 take 8.76. *Ans.* 79.186.
7. 1 take .0001. *Ans.* 0.9999.
8. 93 hund'ths take 175 thous'ths. *Ans.* .755.
9. 23 *and* 5 tenths take 12 *and* 176 millionths. *Ans.* 11.499824.
10. 184 *and* 7 thous'ths take 137 and 68 thous'ths. *Ans.* 46.939.
11. 1 thousand take 1 thousandths. *Ans.* 999.999.
12. 7 hunds. take 7 hund'ths. *Ans.* 699.93.

Note.—In the following, first reduce the fractions to decimals, and then subtract.

What is the value of:

13. $8\frac{3}{4} - 7\frac{1}{2}$? *Ans.* 1.25.
14. $\frac{7}{8} - \frac{3}{5}$? *Ans.* 0.275.
15. $1\frac{2}{5} - \frac{3}{4}$? *Ans.* 0.65.
16. $\frac{7}{20} - \frac{3}{32}$? *Ans.* .25625.
17. $\frac{3}{16} - \frac{7}{400}$? *Ans.* .17.
18. $24\frac{3}{8} - 13\frac{1}{20}$? *Ans.* 11.325.

Multiplication of Decimals.

219. 1. Multiply .9 by .07.

OPERATION.

.9
.07
.063 *Ans.*

Explanation. — $.9 = \frac{9}{10}$, and $.07 = \frac{7}{100}$.
Now by Art. **194**, $\frac{9}{10} \times \frac{7}{100} = \frac{63}{1000} = .063$.

Hence, the

Rule.—*Multiply as in whole numbers, and in the product point off as many decimal figures from the right as there are decimal places in both factors, prefixing ciphers when necessary to supply the deficiency.*

Exercises.

Multiply:

2. 4.5 by 6.4. *Ans.* 28.8.
3. 7.08 by 3.2. *Ans.* 22.656.
4. 16.5 by .008. *Ans.* 0.132.
5. 125.08 by .25. *Ans.* 31.27.
6. .0061 by .05. *Ans.* 000305.
7. 122 by .78.
8. 3.25 by 16.
9. 4.508 by .24.
10. .1806 by 5.4.
11. .0586 by .75.

12. The distance around a circle is 3.1416 times the distance through it; how far is it around a circular garden if it is 75 yards through it? *Ans.* 235.62 yards.

Division of Decimals.

220. Since Division is the *reverse* of Multiplication, by reversing the rule of the latter we get the

Rule.—*Divide as in whole numbers, and in the quotient*

point off as many decimal figures from the right as the decimal places of the dividend exceed those of the divisor, prefixing ciphers, if necessary, to supply the deficiency

NOTES. — I. When there are more decimal places in the divisor than in the dividend, make them equal by annexing ciphers to the dividend before dividing.

II. If there is a remainder, ciphers may be annexed to it as decimals, and the division continued at pleasure.

III. When there is a remainder at the close of the operation, the sign + should be annexed to the quotient to show that it is not complete.

EXERCISES.

Divide:

1. 177.6 by 2.4. *Ans.* 74.
2. 62.5 by .25. *Ans.* 250.
3. 8.84 by 3.4. *Ans.* 2.6.
4. 3.139 by .43. *Ans.* 7.3.
5. 283.25 by 2.5. *Ans.* ?
6. .0639 by .09. *Ans.* ?
7. 45.625 by 12.5. *Ans.* ?
8. 23421. by 2.11. *Ans.* ?
9. 42.81 by .346. *Ans.* 123.728 +.
10. 12.82561 by 3.01. *Ans.* 4.261.
11. 983 by 6.6. *Ans.* 148.939 +.

221. QUESTIONS FOR REVIEW

What is: 1. A Decimal Fraction? 2. The decimal point? 3. A mixed number?

How do decimals decrease from left to right? On what does the value of a decimal figure depend? The first place on the right of the decimal is that of what? The second place on the right? The third? The fourth? The fifth?

Is the denominator of a decimal written? What is it?

How are decimals read? Repeat the three principles, p. 168.

How are: 1. Decimals reduced to fractions in their lowest terms? 2. Common fractions reduced to decimals?

How are decimals: 1. Added? 2. Subtracted? 3. Multiplied? 4. Divided?

United States Money.

Coins.

Gold Coins.—*Eagle, half eagle, quarter eagle, three dollar* piece, and *dollar.*

Silver Coins.—*Dollar, half dollar, quarter dollar, dime.*

Nickel and Bronze Coins.—5-*cent*, 3-*cent*, and 1-*cent* pieces.

222. The **Currency** of a nation means its money.

223. U. S. Money is the legal currency of the United States. Its denominations are Eagles (E.), Dollars ($), Dimes (d.), Cents (c.), and Mills (m.).

TABLE.

10 m. = 1 c.	**E.**	**$.**	**d.**	**c.**	**m.**
10 c. = 1 d.	1 =	10 =	100 =	1000 =	10000
10 d. = $1.		1 =	10 =	100 =	1000
$10 = 1 E.			1 =	10 =	100

224. The U. S. coins are *gold*, *silver*, *nickel*, and *bronze*.

In addition to the coins, p. 174, is the double eagle (gold).

The weight of the gold dollar is 25.8 grains, and that of the silver dollar is 412½ grains.

225. The Dollar is the **Unit**, and the only denominations used in practice are *dollars* and *cents*. Cents are hundredths, and mills thousandths of a dollar.

Hence, dollars are written with the sign ($) prefixed to them, and the point (.) placed after them, and cents and mills are written in the hundredths and thousandths places respectively on the right of the point.

Thus, we write 12 dollars 23 cents and 6 mills, $12.236;
12 dollars 4 cents and 3 mills, $12.043;
12 dollars and 7 mills, $12.007.

All vacant places are filled with 0's.

REDUCTION OF U. S. MONEY.

MENTAL EXERCISES.

226. How many cents in:

1. 1 dime? 5 d.? 7 d.? 10 d.? 23 d.? ½ d.? 2½ d.?
2. 1 dollar? $3? $8? $10? $67? $½? $¼? $¾?
3. 10 mills? 20 m.? 40 m.? 100 m.? 130 m.?

How many dimes in:

4. 1 dollar? $7? $9? $45? $\$\frac{1}{2}$? $\$5\frac{1}{2}$? $\$\frac{3}{10}$? $\$6\frac{7}{10}$?

5. 1 cent? 30 c.? 170 c.? 43 c.? (*Ans.* 4.3 c.) 75 c.? 112 c.? 5 c.?

How many mills in:

6. 1 cent? 7 c.? 31 c.? 125 c.? 83 c.? $\frac{1}{2}$ c.? $4\frac{1}{2}$ c.? 10 c.?

7. 1 dime? 2 d.? 5 d.? 17 d.? $\frac{1}{2}$ d.? $2\frac{1}{2}$ d.? $\frac{1}{10}$ d.? $\frac{7}{10}$ d.?

8. 1 dollar? $3? $12? $\$\frac{1}{2}$? $\$\frac{1}{10}$? $\$\frac{9}{10}$? $\$\frac{1}{8}$? $\$\frac{1}{100}$?

227. CASE I.—From a higher to a lower denomination.

1. Reduce $5 to cents; also to mills.

In $1 there are 100 c.; hence, in $5 there are 5 times 100 c. = 500 cents.

In $1 there are 1000 m.; hence, in $5 there are 5 times 1000 m. = 5000 mills.

OPERATION.

5×100 c. $= 500$ c.

5×1000 m. $= 5000$ m.

Hence, the

RULE.—I. *To reduce dollars to cents, multiply by* 100 *or annex two ciphers.*

II. *To reduce dollars to mills, multiply by* 1000, *or annex three ciphers.*

EXERCISES.

Reduce:

2. $7 to cents. *Ans.* 700 c.

3. $19 to mills. *Ans.* 19000 m.

4. $125 to mills. *Ans.* ?

5. $11 to cents. *Ans.* ?

6. $162 to mills. *Ans.* ?

7. $3274 to cents. *Ans.* ?

How many:

8. Cents in $\$\frac{1}{2}$? *Ans.* 50.

9. Mills in $\$\frac{3}{4}$? *Ans.* 750.

10. Cents in $\$\frac{3}{5}$? *Ans.* ?

11. Mills in $\$\frac{5}{8}$? *Ans.* 625.

12. Cents in $\$\frac{17}{25}$? *Ans.* ?

13. Mills in $\$\frac{11}{20}$? *Ans.* ?

228. CASE II.—From a lower to a higher denomination.

RULE.—*To reduce cents or mills to dollars, reverse the rule in Art.* **227,** *and divide by* 100 *or* 1000, *or point off two or three figures from the right for decimals.*

EXERCISES.

1. Reduce 375 cents to dollars. *Ans.* $3.75.
2. Reduce 4261 mills to dollars. *Ans.* $4.261.

How many dollars in:

3. 37 cents? *Ans.* .37.
4. 421 mills? *Ans.* .421.
5. 679 cents? *Ans.* ?
6. 8743 mills? *Ans.* ?
7. 75 cents? *Ans.* $\frac{3}{4}$.
8. 875 mills? *Ans.* $\frac{7}{8}$.

EXERCISES IN U. S. MONEY.

229. Addition, Subtraction, Multiplication, and Division of U. S. Money are evidently performed according to the rules of decimal fractions.

230. 1. Add together $473.43, $530.75, $645.29, $432.19, $5663.25. *Ans.* $7744.91.

2. What is the number whose **c** parts are $45, $37.50, $18.75, $45.25, and $12.25? *Ans.* $158.75.

3. A farmer sold a lot of cotton for $1235, a lot of corn for $526.35, a load of peas for $37.245, a yoke of oxen for $42, and a heifer for $18.235; what was the total amount? *Ans.* $1858.83.

4. From $327.59 take $163.75. *Ans.* $163.84.

5. Find the difference between $545 and $275.25. *Ans.* $269.75.

6. A owes B $325.05 and B owes A $284.785; how should they settle? *Ans.* A should pay B $40.265.

7. The multiplicand is \$434.25, the multiplier is 3.075; what is the product? *Ans.* \$1335.31875.

8. What is the value of 9 bales of cotton, averaging 450.6 pounds a bale, and worth 8.5 cents per pound? *Ans.* \$344.709.

9. A farmer sold 8.6 barrels of syrup, averaging 31.23 gallons a barrel, at 62.5 cents per gallon; what did he receive for all? *Ans.* \$167.86+.

10. What is the quotient of \$984.15 by 243? *Ans.* \$4.05.

11. The dividend is \$63.25, the divisor 2.7; what is the quotient? *Ans.* \$23.42+.

12. A farmer sold 6.4 acres for \$148.992; how much was that per acre? *Ans.* ?

13. Among how many persons can \$197.4 be distributed if each person receives \$1.128? *Ans.* 175.

14. A merchant bought 125 barrels of apples at \$3.50, and sold 40 barrels at \$3.25, and the remainder at \$4.10 a barrel; how much did he gain or lose by the operation? *Ans.* \$41 *gain.*

15. A farmer bought 160 hogs at \$4.25 a head, 18 sheep at \$6.50 a head, two wagons at \$87.45 apiece, and paid in cash \$604.25; how much did he then owe? *Ans.* \$367.65.

Accounts and Bills.

231. A Debt is money, goods, or services due from one party to another. A *debtor* is a person who owes a debt, a *creditor* one to whom a debt is due.

232. A Bill of Goods is a written statement given by

the seller to the buyer, containing the date of the purchase, the names of the buyer and seller, a list of the goods bought, with their prices, and the total amount.

An Item is any article in the bill; *extending* an item is finding its cost, and the *footing* is the entire cost of all the items in a bill.

233. An Account is a written statement of debits and credits between two parties.

When a bill or account is paid, the creditor should write at the bottom of the same: *Received payment*, and after it, his name.

The symbol @ stands for *at*.

Bills.

334. Extend the items and find the footings of the following bills:

(1)

Louisville, *June 23, 1885.*

Dr. W. M. Baker,

Bought of George Coleman.

5 yards broadcloth @ $3.25		
3 yards cambric @ $.12½		
3 dozen buttons @ $.15		
6 Skeins sewing silk @ $.06¼		
4 yards wadding @ $.08		
Amount	17	77

Received payment,

George Coleman.

(**2**)

BATON ROUGE, *June 16, 1885.*

Mr. J. R. Holmes,

Bought of Wm. Garig & Co.

86 pounds coffee @ 10¼ c.....		
38 pounds tea @ 85 c.....		
63 gallons molasses @ 37½ c.....		
125 pounds rice @ 7 c.....		
75 pounds starch @ 4 c.....		
56 pounds bar-soap @ 5¼ c.....		
Amount.....	79	43

Received payment,

Wm. Garig & Co.

235. QUESTIONS FOR REVIEW.

What is U. S. money? What are the denominations? Repeat the table.

What are: 1. The gold coins? 2. The silver coins? 3. The nickel coins? 4. The bronze coins?

What is the unit? What denominations only are used in practice? How are: 1. Dollars expressed? 2. Cents expressed? 3. Mills expressed?

How are dollars reduced: 1. To cents? 2. To mills?

How are: 1. Cents reduced to dollars? 2. Mills reduced to dollars?

How are the operations of Addition, Subtraction, Multiplication, and Division of U. S. money performed?

What is: 1. A debt? 2. A debtor? 3. A creditor? 4. A bill of goods? 5. An item? 6. An account?

What is meant by: 1. Extending an item? 2. Finding the footing of a bill?

When a bill or an account is paid, what should be done?

Compound Numbers.

Definitions.

236. A Measure is a standard unit of quantity by which similar quantities are measured, and their amounts or values estimated; as 1 pound, 1 hour, 1 dime, etc.

237. A Simple Number is an amount expressed in terms of *one* measure; as 5 feet, 9 pounds.

238. A Compound Number is an amount expressed in terms of different, but similar, measures; as 5 yards 2 feet 7 inches, 9 weeks 5 days. 7 yards 3 days is not a compound number, as the measures, *yard* and *day*, are not similar things.

Linear Measure.

239. The Measures used in measuring distances, as length, width, and height, are the *mile* (**mi.**), the *chain* (**ch.**), the *rod* (**rd.**), the *yard* (**yd.**), the *foot* (**ft.**), and the *inch* (**in.**).

Table.

12 *in.* = 1 *ft.**	4 *rd.* = 1 *ch.*
3 *ft.* = 1 *yd.*	80 *ch.* = 1 *mi.*
$5\frac{1}{2}$ *yd.* = 1 *rd.*	

* The table is read: 12 inches are 1 foot, 3 feet are 1 yard, etc.

Mental Exercises.

240. 1. How many inches in 2 feet? 5 feet? 7 feet? $\frac{1}{2}$ foot? $\frac{1}{3}$ foot?

2. How many feet in 36 inches? 60 in.? 108 in.? 30 in.? 6 in.?

3. How many feet in 2 yards? 9 yds.? 20 yds.? $\frac{1}{2}$ yd.? $\frac{1}{3}$ yd.?

4. How many yards in 9 feet? 11 feet? 10 rd.? 36 in.? 72 in.?

5. How many rods in 3 chains.? 5 ch.? 7 ch.? $\frac{1}{2}$ ch.? 11 yd.?

6. How many chains in 2 miles? $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{10}$ mi.?

Written Exercises.

241. Reduction is the process of changing the measures of numbers without changing their amounts. It is of two kinds, viz: *Descending* and *Ascending.*

242. I. **Reduction Descending** is changing from a higher to a lower measure.

1. Reduce 8 yd. 2 ft. 7 in. to inches.

Explanation.—Since 1 yd.=3 ft., we multiply 8 by 3 to reduce the yds. to ft., and add in the 2 ft., making 26 ft. Again, since 1 ft. =12 in., we multiply 26 by 12 to reduce the ft. to in., and add in the 7 in., making 319 inches.

OPERATION.

8 yd. 2 ft. 7 in.
3
26
12
319

243. Hence, for reduction descending, we have the

Rule I.—*Multiply the number of the highest measure by the number required of the next lower measure to make one*

of the higher, and to the product add the number of the lower measure, if any.

II. *Proceed in like manner with the result, and so continue until the required measure is reached.*

244. II. **Reduction Ascending** is changing from a lower to a higher measure.

2. Reduce 319 in. to yards.

EXPLANATION.—Since 1 ft. = 12 in., we divide the 319 in. by 12 to reduce to ft., and obtain 26 ft. and 7 in. over. Again, since 1 yd. = 3 ft., we divide 26 ft. by 3 to reduce to yd., and get 8 yd. and 2 ft. over. We thus obtain 8 yd. 2 ft. 7 in., *Ans.*

OPERATION.

12)319 in.

3)26 ft. 7 in.

8 yd. 2 ft.

245. Hence, for reduction ascending, we have the

RULE I.—*Divide the number by the number required of its measure to make one of the next higher.*

II. *In the same manner divide the quotient, and so on, until the required measure is reached. The last quotient with the remainders annexed, will be the required result.*

Reduce:

3. 10 yds. 2 ft. 4 in. to inches. *Ans.* 388 in.
4. 139 in. to yards. *Ans.* 3 yd. 2 ft. 7 in.
5. 6 mi. 22 ch. 2 rd. to rods. *Ans.* 2010 rd.
6. 8 ch. 2 rd. 3 yd. to yards. *Ans.* 190 yd.
7. 763 rd. to miles. *Ans.* 2 mi. 30 ch. 3 rd.
8. 16 rd. 2 ft. to feet. *Ans.* 266 ft.
9. 8375 in. to rd. *Ans.* ?
10. 20 mi. 12 ch. 2 yd. 5 in. to inches. *Ans.* ?

SQUARE MEASURE.

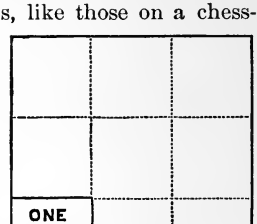
246. **A Surface** is that which has only two dimen-

sions: *length* and *width;* as the *face* of a black-board, or the *surface* of a floor or slate.

All surfaces are measured by squares, like those on a chess-board, or like this figure:

A square inch is a surface 1 inch long and 1 inch wide.

A square foot is a surface 1 foot long and 1 foot wide.



A Square Yard.

The measures used in measuring surfaces are the *square mile* (**sq. mi.**), the *acre* (**A.**), the *square chain* (**sq. ch.**), the *square rod* (**sq. rd.**), the *square yard* (**sq. yd.**), the *square foot* (**sq. ft.**), and the *squre inch* (**sq. in.**)

Table.

144 **sq. in.** = 1 **sq. ft.**	16 **sq. rd.** = 1 **sq. ch.**
9 **sq. ft.** = 1 **sq. yd.**	10 **sq. ch.** = 1 **A.**
$30\frac{1}{4}$ **sq. yd.** = 1 **sq. rd.**	640 **A.** = 1 **sq. mi.**

Measures sometimes used: 1 **sq. rd.** = 1 perch or pole (**P.**); 40 **P.** = 1 rood (**R.**); 4 **R.** = 1 acre.

Mental Exercises.

247. 1. How many sq. in. in 2 sq. ft.? 5 sq. ft.? $\frac{1}{2}$ sq. ft.? $\frac{1}{4}$ sq. ft.?

2. How many sq. ft. in 144 sq. in.? 432 sq. in.? 72 sq. in.? 36 sq. in.?

3. How many sq. ft. in 3 sq. yd.? 9 sq. yd.? 20 sq. yd.? $\frac{1}{3}$ sq. yd.?

4. How many sq. rd. in 5 sq. ch.? 7 sq. ch.? $\frac{1}{2}$ sq. ch.? $\frac{1}{4}$ sq. ch.?

5. How many acres in 40 sq. ch.? 100 sq. ch.? 75 sq. ch.? 2 sq. ch.?

Written Exercises.

248. Reduce:

1. 9 sq. yd. 7 sq. ft. to square feet. *Ans.* 88 sq. ft.
2. 93 sq. ft. to square yards. *Ans.* 10 sq. yd. 3 sq. ft.
3. 84 sq. rd. 12 sq. yd. 6 sq. ft. to square feet. *Ans.* 22983 sq. ft.
4. 583 sq. rd. to acres. *Ans.* 3 A. 6 sq. ch. 7 sq. rd.
5. 17 A. 3 R. 15 P. to poles or perches. *Ans.* 2855 P.
6. 13573 sq. ch. to square miles. *Ans.* 2 sq. mi. 77 A. 3 sq. ch.
7. 16725 sq. in. to square yards. *Ans.* 12 sq. yd. 8 sq. ft. 21 sq. in.
8. 16 sq. rd. 5 sq. ft. to square inches. *Ans.* ?
9. 11 A. 8 sq. ch. 3 sq. yd. to square feet. *Ans.* ?
10. 1 sq. mi. to inches. *Ans.* ?

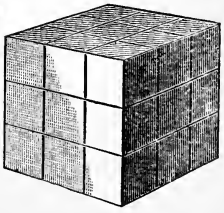
Solid or Cubic Measure.

249. A Volume or **Solid** is that which has three dimensions: length, width and height; as a box, a room, or a book.

All volumes are measured by cubes like this figure:

A cubic inch is a volume 1 in. long, 1 in. wide, 1 in. high.

A cubic foot is a volume 1 ft. long, 1 ft. wide, 1 ft. high.



A Cubic Yard.

The measures used in measuring volumes are the *cord* (**c.**), the *cubic yard* (**cu. yd.**), the *cubic foot* (**cu. ft.**), and the *cubic inch* (**cu. in.**)

TABLE.

1728 **cu. in.** = 1 **cu. ft.**	128 **cu. ft.** = 1 **c.**
27 **cu. ft.** = 1 **cu. yd.**	

A cord is used for measuring wood. A pile of wood 8 feet long, 4 feet wide, and 4 feet high is a cord. One foot in length of this pile, or 16 cu. ft., is a cord foot.

MENTAL EXERCISES.

250. 1. How many cu. ft. in 2 cu. yd.? 5 cu. yd.? $\frac{1}{3}$ cu. yd.? $\frac{1}{9}$ cu. yd.?

2. How many cu. yd. in 81 cu. ft.? 270 cu. ft.? 9 cu. ft.? 3 cu. ft.?

WRITTEN EXERCISES.

251. Reduce:

1. 13 cu. ft. to cubic inches. *Ans.* 22464 cu. in.
2. 25 cu. ft. 524 cu. in. to cubic inches. *Ans.* 43724 cu. in.
3. 2379 cu. in. to cu. ft. *Ans.* 1 cu. ft. 651 cu. in.
4. 9 cu. yd. 123 cu. in. to cubic inches. *Ans.* 420027 cu. in.
5. 1274 cu. ft. to cords. *Ans.* ?

MEASURES OF CAPACITY.

Capacity means amount of bulk or space.

I. LIQUID MEASURE.

252. The Measures used in measuring liquids, such as water, milk, oil, whisky, etc., are the *hogshead* (**hhd.**), the

barrel (**bar.** or **bbl.**), the *gallon* (**gal.**), the *quart* (**qt.**), the *pint* (**pt.**), and the *gill* (**gi.**).

GI. PT. QT. GAL. BAR. HHD.

TABLE.

4 **gi.** = 1 **pt.**	31½ **gal.** = 1 **bar.**
2 **pts.** = 1 **qt.**	63 **gal.** = 1 **hhd.**
4 **qts.** = 1 **gal.**	

One gallon contains 231 cu. inches.

MENTAL EXERCISES.

253. How many:

1. Gills in 2 pt.? 5 pt.? 1 qt.? 3 qt.? 1 gal.? 2 gal.? ½ gal.?
2. Pints in 8 gi.? 32 gi.? 2 qt.? 7 qt.? ½ qt.? 3 gal.? ½ gal.?
3. Quarts in 4 pt.? 20 pt.? 2 gal.? 10 gal.? ½ gal? ¼ gal.? 32 gi.?
4. Gallons in 1 bar.? 2 bar.? 2 hhd.? 20 qt. 56 pt.? 64 gi? ⅓ hhd.?

WRITTEN EXERCISES.

254. Reduce:

1. 7 gal. 2 qt. 1 pt. to pints. *Ans.* 61 pt.
2. 1039 gi. to gallons. *Ans.* 32 gal. 1 qt. 1 pt. 3 gi.
3. 32 gal. 2 qt. 1 pt. to gills. *Ans.* 1044 gills.

4. 367 pt. to gallons. *Ans.* 45 gal. 3 qt. 1 pt.
5. 4625 qt. to hogsheads. *Ans.* 18 hhd. 22 gal. 1 qt.
6. 3 hhd. 17 gal. 1 pt. to gills. *Ans.* 6596 gi.
7. 10 bar. 3 gi. to gills. *Ans.* 10083 gi.
8. 3 hhd. 1 bar. $16\frac{1}{2}$ gal. to pints. *Ans.* 1896 pt.
9. 1024 pt. to barrels. *Ans.* ?
10. 1 hhd. to gills. *Ans.* ?

II. Dry Measure.

255. The measures used in measuring quantities that are not liquid, such as grain, potatoes, coal, etc., are the *bushel* (**bu.**), the *peck* (**pk.**), the *quart* (**qt.**), and the *pint* (**pt.**).

Table.

2 pt. = 1 qt. **8 qt. = 1 pk.** **4 pk. = 1 bu.**

One *bushel* contains 2150.4 cu. inches; hence, one *gallon*, Dry Measure, contains 268.8 cu. inches.

Mental Exercises.

256. How many:

1. Pints in 2 qt.? 7 qt.? $\frac{1}{2}$ qt.? 1 pk.? 5 pk.? $\frac{1}{2}$ pk.?
2. Quarts in 8 pt.? 20 pt.? 11 pt.? 2 pk.? 7 pk.? $\frac{1}{4}$ pk.?
3. Pecks in 16 qt.? 36 qt.? 16 pt.? 48 pt.? 2 bu.? $\frac{1}{2}$ bu.?

Written Exercises.

257. Reduce:

1. 2 pk. 3 qt. 1 pt. to pints. *Ans.* 39 pt.
2. 10 bu. 3 pk. 7 qt. to quarts. *Ans.* 351 qt.
3. 57 pt. to pecks. *Ans.* 3 pk. 4 qt. 1 pt.
4. 195 qt. to bushels. *Ans.* 6 bu. 3 qt.
5. 34 bu. 5 qt. to pints. *Ans.* ?
6. 1765 pt. to bushels. *Ans.* ?
7. 21 bu. 3 pk. 5 qt. 1 pt. to pints. *Ans.* ?

Measures of Weight.

I. Troy Weight.

258. The measures used in weighing precious stones, gold, silver, etc., are the *pound* (**lb.**), the *ounce* (**oz.**), the *pennyweight* (**pwt.**), and the *grain* (**gr.**).

Table.

24 **gr.** = 1 **pwt.** 20 **pwt.** = 1 **oz.** 12 **oz.** = 1 **lb.**

Mental Exercises.

259. How many:

1. Grains in 2 pwt.? 5 pwt.? $\frac{1}{2}$ pwt.? $\frac{1}{3}$ pwt.? $\frac{1}{4}$ pwt.?
2. Pennyweights in 72 gr.? 240 gr.? 3 oz.? $\frac{1}{2}$ oz.? $\frac{1}{5}$ oz.?
3. Ounces in 40 pwt.? 70 pwt.? 5 lb.? $\frac{1}{3}$ lb.? $\frac{1}{4}$ lb.?

Written Exercises.

260. Reduce:

1. 2 oz. 7 pwt. 19 gr. to grains. *Ans.* 1147 gr.
2. 5 lb. 10 oz. 13 pwt. to pennyweights. *Ans.* 1413 pwt.
3. 8 lb. 16 pwt. to grains. *Ans.* 46364 gr.
4. 245 pwt. to pounds. *Ans.* 1 lb. 5 pwt.
5. 677 gr. to ounces. *Ans.* 1 oz. 8 pwt. 5 gr.
6. 8493 gr. to pounds. *Ans.* ?
7. 205 oz. 12 gr. to grains. *Ans.* ?

II. Avoirdupois Weight.

261. The measures used in weighing articles, such as hay, cotton, groceries, etc., are the *ton* (**t.**), the *hundredweight* (**cwt.**), the *pound* (**lb.**), the *ounce* (**oz.**), and the *dram* (**dr.**).

Table.

16 **dr.** = 1 **oz.**	100 **lb.** = 1 **cwt.**
16 **oz.** = 1 **lb.**	20 **cwt.** = 1 **t.**

Mental Exercises.

262. How many:

1. Drams in 2 oz.? 4 oz.? $\frac{1}{2}$ oz.? $\frac{1}{4}$ oz.? $\frac{1}{8}$ oz.? $1\frac{1}{2}$ oz.?
2. Which is the lowest measure? The next lowest? etc.
3. Which is the highest measure? The next highest? etc.

Reduce:

4. 3 t. to cwt.; 5 cwt. to lb.; 10 lb. to oz.; 20 oz. to dr.
5. 100 cwt. to t.; 700 lb. to cwt.; 320 oz. to lb.; 160 dr. to oz.

Written Exercises.

263. Reduce:

1. 2 lb. 5 oz. 10 dr. to drams. *Ans.* 602 dr.
2. 5 cwt. 80 lb. 12 oz. to ounces. *Ans.* 9292 oz.
3. 7 t. 17 cwt. 50 lb. to pounds. *Ans.* 15750 lb.
4. 5285 pounds to tons. *Ans.* 2 t. 12 cwt. 85 lb.
5. 4364 oz. to hundredweights. *Ans.* 2 cwt. 72 lb. 12 oz.
6. 25607 dr. to pounds. *Ans.* ?
7. 5 t. 10 cwt. 10 lb. 12 oz. to drams. *Ans.* ?
8. 512257 dr. to tons. *Ans.* 1 t. 1 lb. 1 dr.

III. Apothecaries' Weight.

264. The measures used in mixing medicines are the *pound* (**lb.** or ℔), the *ounce* (**oz.** or ℥), the *dram* (**dr.** or ʒ), the *scruple* (**scr.** or ℈), and the *grain* (**gr.**).

Table.

20 **gr.** = 1 **scr.**	8 **dr.** = 1 **oz.**
3 **scr.** = 1 **dr.**	12 **oz.** = 1 **lb.**

Note. — The pound, the ounce, and the grain are the same measures in Troy and Apothecaries' weight.

Mental Exercises.

265. How many:

1. Grains in 3 scr.? 5 ℈? 7 scr.? 10 ℈? ½ scr.? ⅕ ℈?
2. Which is the lowest measure? The next lowest? etc.
3. Which is the highest measure? The next highest? etc.

Written Exercises.

266. How many:

1. Grains in 5 oz. 6 dr. 2 scr. 10 gr.? *Ans.* 2810 gr.
2. Scruples in 3 lb. 10 oz. 5 dr. 1 scr.? *Ans.* 1120 scr.
3. Drams in 5 lb. 4 ℥ 2 ʒ? *Ans.* 514 ʒ.
4. Scruples in 12 lb. 7 dr.? *Ans.* 3477 scr.
5. Pounds, etc., in 99 dr.? *Ans.* 1 lb. 3 dr.
6. Ounces, etc., in 167 scr.? *Ans.* 6 oz. 7 dr. 2 scr.
7. Drams, etc., in 583 gr.? *Ans.* 9 dr. 2 scr. 3 gr.
8. Pounds, etc., in 564307 grains? *Ans.* ?
9. Grains in 5 lb. 5 dr. 5 gr.? *Ans.* ?

Measures of Money.

267.—I. United States Money.

Note.—For table and exercises under this head, see United States Money, page 174.

II. English Money.

268. English or Sterling money is the currency of Great Britain. The denominations, or measures, are the *pound* (**£.**), the *shilling* (**s.**), the *penny* (**d.**), and the *farthing* (**far.**).

Table.

4 **far.** = 1 **d.** 12 **d.** = 1 **s.** 20 **s.** = 1 **£.**

Note.—A florin = 2 s.; a guinea = 21 s.; and 1 £ = $4.84.

Mental Exercises.

269. How many:

1. Farthings in 3 d.? 5 d.? $\frac{1}{2}$ d.? $\frac{1}{4}$ d.?
2. Pence in 20 far.? 30 far.? 2 s.? 6 s.? $\frac{1}{2}$ s.?
3. Shillings in 24 d.? 72 d.? 3 £? $\frac{1}{2}$ £? $\frac{1}{5}$ £?
4. Which is the lowest denomination? The next? etc.
5. Which is the highest denomination? The next? etc.

Written Exercises.

270. Reduce:

1. 5 £. 4 s. 10 d. to pence. *Ans.* 1258 d.
2. 16 s. 5 d. 3 far. to farthings. *Ans.* 791 far.
3. 7 s. 1 far. to farthings. *Ans.* 337 far.
4. 251 d. to pounds. *Ans.* 1 £. 11 d.
5. 100 far. to shillings. *Ans.* 2 s. 1 d.
6. 793 s. to pounds. *Ans.* 39 £. 13 s.
7. 10 £. 3 d. to farthings. *Ans.* ?
8. 53675 far. to pounds. *Ans.* ?

271.—III. French Money.

Table.

10 milliemes (*mĭ-lamé*) = 1 centime.
100 centiemes (*sŏn-teem'*) = 1 franc.

Note.—One franc is equal to $.186 U. S. money.

Measure of Time.

272. The Units, or Measures, used in measuring time, are the *century* (**C.**), the *year* (**yr.**), the *month* (**mo.**), the *week* (**wk.**), the *day* (**d.** or **da.**), the *hour* (**hr.** or **h.**), the *minute* (**m.**), and the *second* (**sec.** or **s.**).

TABLE.

60 s. = 1 m.	7 da. = 1 wk.
60 m. = 1 h.	365 da. = 1 yr.
24 h. = 1 da.	

The Solar Year is exactly 365 da. 5 hr. 48 m. 49.7 sec., or $365\frac{1}{4}$ days nearly. In four years this fraction amounts nearly to one day. To provide for this excess one day is added to the month of February every fourth year, which is called **Leap Year** (L. yr.).

Every year, except those ending with two 0's, that is exactly divisible by 4 is a L. yr.; as 1844, 1856, 1884.

Every year ending with two 0's that is exactly divisible by 400 is a L. yr.; as 1600, 2000, 2400.

Every year which is not so divisible is a common year; as 1847, 1855, 1900, 1800.

A common year consists of 365 days, a leap year of 366 days, and a century of 100 successive years.

The Civil Year is divided into twelve Calendar months, thus:

January (Jan.)	1st mo...31 da.	July (July)	7th mo. 31 da.
February (Feb.)	2d mo...28 da.	August (Aug.)	8th mo. 31 da.
March (Mar.)	3d mo...31 da.	September (Sep.)	9th mo. 30 da.
April (Apr.)	4th mo...30 da.	October (Oct.)	10th mo. 31 da.
May (May)	5th mo...31 da.	November (Nov.)	11th mo. 30 da.
June (June)	6th mo...30 da.	December (Dec.)	12th mo. 31 da.

MENTAL EXERCISES.

273. How many:

1. Seconds in 2 m.? 5 m.? $\frac{1}{2}$ m.? $\frac{1}{4}$ m.? $\frac{1}{10}$ m.?
2. Minutes in 180 s.? 90 s.? 3 hr.? $\frac{1}{2}$ hr.? $\frac{1}{6}$ hr.?
3. Hours in 120 m.? 600 m.? 2 da.? $\frac{1}{2}$ da.? $\frac{1}{3}$ da.?
4. Days in 48 hr.? 36 hr.? 240 hr.? 2 wk.? 5 wk.?
5. Is 1824 a common or a leap year? 1838? 1874? 1855? 1900? 1700? 1600? 1950? 2200? 2800? 3000?

Written Exercises.

274. How many:

1. Days in 32 common years? *Ans.* 11680 da.
2. Days in 32 leap years? *Ans.* 11712 da.
3. Hours in 5 yr. 120 da. 15 hr.? *Ans.* 46695 hr.
4. Hours in 10 L. yr. 106 da. 17 hr.? *Ans.* 90401 hr.
5. Minutes in 3 wk. 5 da. 10 hr. 12 m.? *Ans.* ?
6. Weeks, etc., in 583 hr.? *Ans.* 3 wk. 3 da. 7 hr.
7. Years, etc., in 45375204 m.?

Ans. 86 yr. 120 da. 13 hr. 24 m.

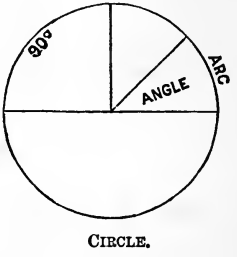
8. Days, etc., in 1000000 sec.? *Ans.* ?
9. How many days are in the century beginning with the year 1801 and ending with the year 1900?

Ans. 36524 da.

Suggestion.—Multiply 365 da. by 100, and to the product add as many days as there are L. yr.

Circular Measure.

275. The measures used in measuring angles and the arcs of circles are the *circle* (**cir.**), the *degree* (°), the *minute* (′), and the *second* (″).



Circle.

Table.

$60'' = 1'$. $60' = 1°$. $360° = 1$ **cir.**

Exercises.

276. How many:

1. Seconds in 7° 15′ 25″? *Ans.* 26125″.
2. Minutes in 1 cir.? *Ans.* 21600′.
3. Minutes in 3 cir. 150° 15′? *Ans.* 73815′.

4. Seconds in 16° 50″? *Ans.* 57650″.
5. Degrees, etc., in 7453″? *Ans.* 2° 4′ 13″.
6. Circles, etc., in 584375′? *Ans.* 27 cir. 19° 35′.
7. Circles, etc., in 73564807″? *Ans.* ?
8. Seconds in 10 cir. 5° 35′ 45″? *Ans.* ?

Paper Measure.

277. The measures used in measuring paper are the *bale* (**b.**), the *bundle* (**bun.**), the *ream* (**rm.**), the *quire* (**qr.**), and the *sheet* (**sht.**).

Table.

24 **sht.** = 1 **qr.**	2 **rm.** = 1 **bun.**
20 **qr.** = 1 **rm.**	5 **bun.** = 1 **b.**

Miscellaneous Table.

12 units = 1 dozen.	4 inches = 1 hand.
12 dozen = 1 gross.	6 feet = 1 fathom.
20 units = 1 score.	8 furlongs = 1 mile.

See, also, Art. **113.**

The Old French Measure.

278. The old French Linear and Land Measure is still partly used in Louisiana, and in other French settlements of the United States.

Table.

12 lines = 1 inch.	6 feet = 1 toise.
12 inches = 1 foot.	32 toises = 1 arpent.

1024 sq. toises = 1 sq. arpent.

The French foot equals 12.79 English inches.

The arpent is the old French name for acre, and is equal to about $\frac{5}{6}$ of an English acre.

Exercises in Reduction.

279. Reduce:

1. 5 ℔. 8 oz. 11 pwt. to grains. *Ans.* 32904 gr.
2. 13 bu. 5 pk. 6 qt. to quarts. *Ans.* 462 qts.
3. 2 mi. 45 ch. to yards. *Ans.* 4510 yd.
4. 5 ch. 3 yd. 2 ft. to inches. *Ans.* 4092 in.
5. 6 yr. 25 da. 6 hr. to minutes. *Ans.* 3189960 m.
6. 12 L. yr. 18 da. to hours. *Ans.* 105840 hr.
7. 25 cu. yd. 15 cu. ft. to cubic feet. *Ans.* 690 cu. ft.
8. 16 sq. rd. 12 sq. yd. 8 sq. ft. to sq. ft.
9. 50 pk. 2 qt. to pints. *Ans.* 804 pt.
10. 18 bar. 10 gal. 2 qt. 1 pt. to pints. *Ans.* 4621 pt.
11. 13° 25′ to seconds. *Ans.* 48300″.
12. 12 £. 5 s. 11 d. to pence. *Ans.* 2951 d.
13. 5 hhd. 15 gal. 1 pt. to gills. *Ans.* 10564 gi.
14. 15 A. 3 R. 20 P. to poles. *Ans.* 2540 P.
15. $2, 5 d. 6 c. to cents. *Ans.* 256c.
16. 7 hunds. 5 tens, 3 ones to ones. *Ans.* 753 ones.
17. 2 ℔. 3 ℥, 4 ʒ, 2 ℈ to scruples. *Ans.* 662 ℈.
18. $43.75 to mills. *Ans.* 47750 m.
19. 5 t. 7 cwt. 74 lb. to pounds. *Ans.* 10774 lb.
20. 7 b. 1 bun. 1 rm. to quires. *Ans.* 1460 qr.

Reduce:

21. 3779 in. to rods. *Ans.* 18 rd. 5 yd. 2 ft. 11 in.
22. 12500 m. to days. *Ans.* 8 d. 16 h. 2 m.
23. 4392 P. to acres. *Ans.* 27 A. 1 R. 32 P.
24. 24352 far. to £., etc. *Ans.* 25 £. 7 s. 4 d.
25. 47643 cu. in. to cu. yds *Ans.* 1 cu. yd. 987 cu. in.
26. 1075 gi. to gallons. *Ans.* 33 gal. 2 qt. 3 gi.
27. 953 ℈ to pounds. *Ans.* 3 ℔. 3 ℥, 5 ʒ, 2 ℈.
28. 895 pt. to bushels. *Ans.* 13 bu. 3 pk. 7 qt. 1 pt.
29. 8433 qrs. to reams. *Ans.* 421 rm. 13 qr.
30. 24563 sq. in. to sq. yds.

Compound Addition.

280. 1. Add together 5 yd. 2 ft. 9 in.; 6 yd. 1 ft. 7 in. and 4 yd. 2 ft. 4 in. *Ans.* 17 yd. 8 in.

Explanation.—Since only like numbers can be added, we write inches under inches, feet under feet, etc. Adding the column of inches, we get 20 in., which, divided by 12, gives 1 ft. 8 in. Set the 8 in. under the column of in., and carry the 1 ft. to the column of ft.; adding this column, we get 6 ft., which equals 2 yd. and 0 ft.; writing 0 under the column of ft., and carrying 2 to the column of yd., we have 17 yds. Hence, the

operation.

yd.	ft.	in.
5	2	9
6	1	7
4	2	4
17	0	8

Rule.—I. *Write the numbers to be added so that those of the same unit may be in the same column.*

II. *Add each column, beginning at the right, divide the sum by the number of units of the column added which equals one of the next higher, set the remainder under that column, and carry the quotient to be added to the next.*

2. What is the sum of 9 £. 16 s. 8 d., and 10 £. 12 s. 7 d.? *Ans.* 20 £. 9 s. 3 d.

3. What is the sum of 7 £. 13 s. 6 d., 2 £. 17 s. 9 d., 3 £. 8 s. 3 d., 9 £. 11 s. 8 d.? *Ans.* 23 £. 11 s. 2 d.

4. What is the sum of 4 bu. 3 pk. 1 qt., 7 bu. 2 pk. 3 qt., 1 bu. 1 pk. 7 qt., and 8 bu.?

Ans. 21 bu. 3 pk. 3 qt.

5. What is the sum of 5 lb. 7 oz. 10 dr., 7 lb. 11 oz., 8 dr., 12 lb. 5 dr., 13 lb., 3 lb. 6 oz. 3 dr?

Ans. 41 lb. 9 oz. 10 dr.

Compound Subtraction.

281. 1. From 7 lb. 5 oz. 9 pwt. 7 gr. take 3 lb. 4 oz. 12 pwt. 4 gr. *Ans.* 4 lb. 17 pwt. 3 gr.

EXPLANATION.– Since only like numbers can be subtracted, we write gr. under gr., pwt. under pwt., etc.

Beginning at the right, we subtract 4 gr. from 7 gr. and get 3 gr., which we write under the column of gr.

OPERATION.

lb.	oz.	pwt.	gr.
7	5	9	7
3	4	12	4
4	0	17	3

Since 12 pwt. is larger than 9 pwt., we take 1 oz. from the 5 oz., leaving 4 oz., and add it, or 20 pwt., to 9 pwt., making 29 pwt. 12 pwt. from 29 pwt. leave 17 pwt.; which we write under the pwt.

Since 1 oz. was taken from 5 oz., we subtract 4 oz. from 4 oz. and get 0 oz., which we write under oz. 3 lb. from 7 lb. leaves 4 lb., which we write under lb.

Hence (see Art. **87**), the

RULE.—I. *Write the less number under the greater so that those of the same kind shall be in the same column.*

II. *Begin at the right and subtract each term from the one above it, if the latter is the greater, and place the difference under the numbers subtracted.*

III. *If any term is greater than the one above it, add to the one above the number of units of that column which equals one of the next higher, from the sum subtract the lower term, write the remainder below, and carry one to the next term to be subtracted, and so on with all the columns.*

2. From 45 A. 2 R. 17 P. take 19 A. 3 R. 36 P.

Ans. 25 A. 2 R. 21 P.

3. From 65 cu. yd. 20 cu. ft. 1252 cu. in. take 55 cu. yd. 26 cu. ft. 956 cu. in.

Ans. 9 cu. yd. 21 cu. ft. 296 cu. in.

4. From 85 bu. 2 pk. take 45 bu. 1 pk. 6 qt.

Ans. 40 bu. 2 qt.

5. From 5 yr. take 3 yr. 9 mo. *Ans.* 1 yr. 3 mo.

6. From 12 lb. 3 ℥, 1 ʒ, take 5 lb. 7 ℥, 5 ʒ, 2 ℈.

Ans. 6 lb. 7 ℥, 3 ʒ, 1 ℈.

Compound Multiplication

282. Multiply 3 £ 5 s. 9 d. by 7. *Ans.* 23 £ 3 d.

Explanation.—We write the multiplier under the term on the right; multiply each term as in simple numbers, setting down and carrying as in compound addition. Thus, 7×9 d. $= 63$ d., which divided by 12 gives 5 s. 3 d. Write the 3 below, and carry 5 s. 7×5 s. $= 35$ s. and 5 s. $= 40$ s., which divided by 20 gives 2 £ 0 s. Write the 0 below and carry 2 £. 7×3 £ $= 21$ £ and 2 £ $= 23$ £.

operation.

£.	s.	d.
3	5	9
		7
23	0	3

Hence, the

Rule.—*Multiply each term of the multiplicand, beginning at the right, by the multiplier; divide the product by the number of units of the term multiplied which equals one of the next higher; set the remainder under that term, and carry the quotient to be added to the next product.*

2. Multiply 22 A. 3 R. 35 P. by 6.
Ans. 137 A. 3 R. 10 P.

3. Multiply 3 lb. 4 oz. 0 dr. 2 scr. by 4.
Ans. 13 lb. 4 oz. 2 dr. 2 scr.

4. Multiply 13 bu. 2 pk. 1 pt. by 15.
Ans. 202 bu. 2 pk. 7 qt. 1 pt.

5. Multiply 5 lb. 3 oz. 13 dr. by 7.
Ans. 36 lb. 10 oz. 11 dr.

6. How many bushels in 9 bins, each containing 120 bu. 3 pk. 3 qt.? *Ans.* 1087 bu. 2 pk. 3 qt.

Compound Division.

283. 1. Divide 44 bu. 3 pk. 3 qt. by 7.
Ans. 6 bu. 1 qt. 5 pk.

EXPLANATION.—Dividing 44 bu. by 7 we get 6 bu. and 2 bu. over. Write the 6 below, reduce the 2 bu. to pk., and to it add 3 pk., which gives 11 pk. 11 pk. ÷ 7 = 1 pk. and 4 pk. over. Write the 1 below, reduce the 4 pk. to qt., and to it add 3 qt., making 35 qt., which divided by 7 gives 5 qt.; write the 5 below.

OPERATION.

bu.	pk.	qt.
7)44	3	3
6	1	5

Hence, the

RULE.—I. *Place the divisor on the left of the dividend, and divide the left-hand term by it, writing the quotient under that term.*

II. *Reduce the remainder, if any, to the next lower unit, adding in like terms of the dividend, if any, and divide the sum by the divisor; and so on, for the other terms.*

2. Divide 24 lb. 7 oz. 8 pwt. by 2. *Ans.* 12 lb. 3 oz. 14 pwt.

3. Divide 10 hhd. 46 gal. 1 qt. 1 pt. by 7.
Ans. 1 hhd. 33 gal. 2 qt. 1 pt.

4. Divide 11 £ 6 s. 3 d. by 5. *Ans.* 2 £ 5 s. 3 d.

5. Divide 36 lb. 10 oz. 9 dr. by 5. *Ans.* ?

284. PARALLEL PROBLEMS.

1.[m] What will $4\frac{1}{2}$ quarts of plums cost at 6 cents a quart? At 3 cents a pint? At 5 cents a pint?

2. What will 5 hogsheads of molasses cost at 37 cents a gallon? *Ans.* $116.55.

3.[m] If 1 bushel of potatoes cost 80 cents, what is the price of 1 peck?

4. If 12 bushels of potatoes cost $6.48, what will be the value of 1 peck? *Ans.* $13\frac{1}{2}$ c.

5.[m] A boy picked 3 quarts of cherries, and sold them at the rate of 5 cents a pint; how much did he receive?

6. A farmer gathered 24 bu. 3 pk. of peaches, and

sold them at the rate of 3 cents a quart; how much did he receive? *Ans.* $23.76.

7.ᵐ A boy bought half a bushel of chestnuts for 30 c., and sold them at 20 c. a peck; how much did he make?

8. A man bought 1 *seventh* of a hogshead of sugar for $12.60, and retailed it at 20 cents a pint; how much did he make? *Ans.* $1.80.

9.ᵐ How many gills are there in $\frac{3}{4}$ of a gallon?

10. How many pints are there in $\frac{11}{14}$ of a hogshead? *Ans.* 399 pt.

11.ᵐ How many hours in $\frac{3}{8}$ of a day? In $\frac{7}{12}$ of a day?

12. How many minutes in $\frac{5}{6}$ of a day? *Ans.* 1200 mi.

13.ᵐ How far will a man travel in 2 hours, if he goes 5 chains in 1 minute?

14. How far will a man travel in 3 weeks at the rate of 5 miles an hour? *Ans.* 2520 mi.

15.ᵐ What will $\frac{3}{4}$ of a ream of paper cost at 20 cents a quire? At 30 cents a quire?

16. What will $\frac{2}{5}$ of a bale of paper cost at $1\frac{1}{2}$ cents a sheet? *Ans.* $28.80.

17.ᵐ How long will it take a man to travel 100 miles if he goes 5 miles an hour?

18. How long would it take a horse, traveling at the rate of 8 miles an hour, to go around the world, a distance of 25000 miles? *Ans.* 130 da. 5 hr.

19. How long would it take a locomotive to go from the earth to the moon, a distance of 240000 miles, traveling at the rate of 25 miles an hour? *Ans.* 1 yr. 35 da.

20.ᵐ How many times will a wheel 8 feet in circumference turn over in going 120 yards?

21. How often will a wheel 12 feet in circumference turn over in going 5 miles? *Ans.* 2200.

22.ᵐ If a buggy, whose wheels are 12 feet in circum-

ference, goes 400 yards in 10 minutes, how often do the wheels turn over in 1 minute?

23. If a locomotive, whose wheels are 15 feet in circumference, runs at the rate of 45 miles an hour, at what rate do the wheels revolve per minute? *Ans.* 264.

24.^{m} What will it cost to build a fence 2 miles long at \$1 a chain?

25. What will 36 miles of telegraph wire cost at 75 cents a rod? *Ans.* \$4320.

26.^{m} If a lad makes 5 steps in walking a rod, how many steps will he make in going 3 chains?

27. If 20 rails are required to build a fence 1 rod long, how many rails will it take to inclose a field $\frac{1}{4}$ of a mile long and $\frac{1}{8}$ of a mile wide? *Ans.* 2400 rails.

285. Questions for Review.

What is a: 1. Simple number? 2. Compound number? State what each of the following measures are used for, name the measures, and repeat the Table: 1. Linear Measure; 2. Square Measure; 3. Solid or Cubic Measure; 4. Liquid Measure; 5. Dry Measure; 6. Troy Weight; 7. Avoirdupois Weight; 8. Apothecaries' Weight; 9. English Money; 10. Measure of Time; 11. Circular Measure; 12. Paper Measure.

What is the rule for: 1. Reduction Descending? 2. Reduction Ascending?

How many: 1. Cubic inches in a gallon? 2. Cubic inches in a bushel? 3. Dollars in 1 £? 4. Cents in 1 franc? 5. Days in a leap-year? 6. Acres in an arpent?

Which are the: 1. Leap-years? 2. Common years?

Name the number of days in each month.

Important Applications.

286. Case I.—To find the time between two dates.

1. What length of time elapsed from July 10, 1843, to Jan. 4, 1845?

Explanation.—Write the latter or greater date for the minuend, and the earlier for the subtrahend, giving the month its *number* instead of the *name*. Thus, since Jan. is the 1st month, we write 1 under mo. and under it write 7, as July is the 7th month. Now subtract as in compound subtraction, allowing 12 mo. to the yr. and 30 da. to the mo.

OPERATION.

yr.	mo.	da.
1845	1	4
1843	7	10
1	5	24

Find the time from:

2. May 12, 1848, to June 1, 1860. *Ans.* 12 yr. 19 da.

3. June 1, 1861, to Oct. 20, 1872. *Ans.* 11 yr. 4 mo. 19 da.

4. June 15, 1846, to Jan. 10, 1848. *Ans.* 1 yr. 6 mo. 25 da.

5. April 20, 1868, to Aug. 1, 1869. *Ans.* 1 yr. 3 mo. 11 da.

6. Henry was born March 9, 1868, and Harry Sept. 15, 1875; how much older is Henry than Harry? *Ans.* 7 yr. 6 mo. 6 da.

7. A note dated July 15, 1879, was paid May 21, 1882; how long did it run? *Ans.* 2 yr. 10 mo. 6 da.

To what age did the following live:

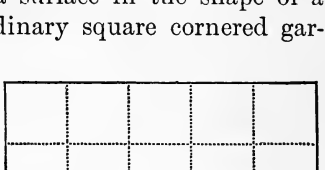
8. Washington, born Feb. 22, 1732; died Dec. 14, 1799?

9. Jefferson, born April 2, 1743; died July 4, 1826?
10. Lincoln, born Feb. 12, 1809; died April 15, 1865?
11. Calhoun, born March 18, 1782; died March 31, 1850?
12. Webster, born Jan. 18, 1782; died Oct. 24, 1852?
13. Clay, born April 12, 1777; died June 29, 1852?

287. CASE II.—To find the area of rectangular surfaces.

A rectangular surface is a surface in the shape of a sheet of paper, or of an ordinary square cornered garden of four sides.

The figure represents a rectangular surface 5 in. long and 3 in. wide, and evidently contains $3\times5=$ 15 square inches. 15 sq. in. is called its *area.*



Hence, the

RULE.—*Multiply the length by the width, expressed in like units.*

1. What is the area of a floor 18 ft. long and 12 ft. wide? *Ans.* 216 sq. ft., or 24 sq. yd.
2. What is the area of a rectangular garden 88 yd. long and 55 yd. wide? *Ans.* 4840 sq. yd., or 1 A.
3. What is the area of a rectangular field 35 ch. long and 24 ch. wide? *Ans.* 840 sq. ch., or 84 A.
4. How many acres in a meadow 17 ch. long and 12 ch. wide? *Ans.* 20.4 A.
5. How many acres in a field 60 rd. long and 44 rd. wide? *Ans.* $16\frac{1}{2}$ A.
6. How many yards of carpeting 3 ft. wide will it take to cover a floor 20 ft. long and 15 ft. wide?

We divide the number of sq. ft., 300, by the width of the carpet, 3 ft., which gives the length of the carpet, 100 ft., and this divided by 3 gives $33\frac{1}{3}$ yd.

OPERATION.

$20 \times 15 = 300$

$300 \div 3 = 100$

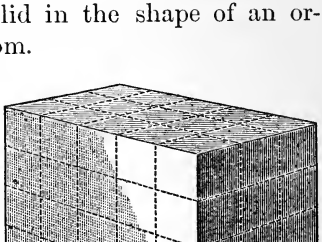
100 ft. $\div 3 = 33\frac{1}{3}$ yd.

7. How many yards of carpeting 4 ft. wide will it take to cover a floor 22 ft. long and 18 ft. wide?

Ans. 33 yd.

288. CASE III.—To find the volume of a rectangular solid.

A rectangular solid is a solid in the shape of an ordinary goods-box, or of a room.



The figure represents a rectangular solid 5 ft. long, 4 ft. high, 3 ft. wide, and evidently contains $5 \times 4 \times 3 = 60$ cu. ft. 60 cu. ft. is called its *volume*. Hence,

RULE.—*Multiply the length, width, and height together.*

NOTE.—The dimensions must be expressed in terms of the same measure.

1. What is the volume of a rectangular box 6 ft. long, 3 ft. wide, and 5 ft. high? *Ans.* 90 cu. ft.

2. What is the volume of a room 20 ft. long, 18 ft. wide, and 12 ft. high? *Ans.* 4320 cu. ft., or 160 cu. yd.

3. How many cubic feet in a marble slab 60 in. long, 24 in. wide, and 6 in. thick? *Ans.* 5 cu. ft.

4. How many cubic yards of coal in a pile 18 ft. long, 10 ft. wide, and 7 ft. high? *Ans.* $46\frac{2}{3}$ cu. yd.

5. How many cubic feet of corn in a crib 20 ft. long, and 12 ft. wide, the corn being $5\frac{1}{2}$ feet deep in the crib?

Ans. 1320 cu. ft.

6. How many cords of wood in a pile 20 ft. long, 12 ft. wide, and 6 ft. high? *Ans.* $11\frac{1}{4}$ cords.

OPERATION. $\frac{20 \times 12 \times 6}{128} = 11\frac{1}{4}$.

How many cords in a pile of wood:

7. 24 ft. long, 8 ft. wide, and 4 ft. high? *Ans.* 6 c.

8. 18 ft. long, 6 ft. wide, and 8 ft. high? *Ans.* $6\frac{3}{4}$ c.

289. CASE IV.—To find the number of gallons in a rectangular box or tank.

1. How many gallons of water will a tank hold which is 6 ft. long, 5 ft. wide, and 8 ft. high?

OPERATION. $6 \times 5 \times 8 \times 7\frac{1}{2} = 1800$ gal. *Ans.*

EXPLANATION.—Since there are 231 cu. in. in one gallon, and 1728 cu. in. in one cubic foot, 1 cu. ft. $= \frac{1728}{231}$ gal. or *about* $7\frac{1}{2}$ gal.

Hence, the

RULE.—*Multiply the number of cu. ft. by* $7\frac{1}{2}$.

2. How many gallons of molasses will a box hold which is 4 ft. long, 2 ft. wide, and 9 in. ($\frac{3}{4}$ ft.) high? *Ans.* 45 gal.

3. A box is 3 ft. long, 2 ft. wide, 8 in. ($\frac{2}{3}$ ft.) high, and is full of honey; how much honey is in the box? *Ans.* 30 gal.

What is the capacity of a tank which is:

4. 6 ft. long, 4 ft. wide, and 10 ft. high? *Ans.* 1800 gal.

5. 4 ft. long, 4 ft. wide, and 9 ft. high? *Ans.* 1080 gal.

6. 5 ft. square at the bottom and 7 ft. high? *Ans.* $1312\frac{1}{2}$ gal.

7. 3 ft. square at the bottom and 6 ft. high?

Ans. 405 gal.

How much milk will a box hold which is:

8. $1\frac{1}{2}$ ft. long, $1\frac{1}{3}$ ft. wide, and $\frac{1}{2}$ ft. high? *Ans.* $7\frac{1}{2}$ gal.

9. $1\frac{2}{3}$ ft. long, $1\frac{1}{5}$ ft. wide, and 6 in. high?

Ans. $7\frac{1}{2}$ gal.

10. 9 in. long, 8 in. wide, and 4 in. deep?

Ans. $1\frac{1}{4}$ gal.

290. CASE V.—To find the number of bushels in a rectangular box, bin, or granary.

1. How many bushels of wheat will a box hold which is 5 ft. long, 4 feet wide, and 3 ft. deep?

OPERATION. $5 \times 4 \times 3 \times .8 = 48.0 = 48$ bu.

EXPLANATION.—Since there are 2150.4 cu. in. in one bushel, and 1728 cu. in. in one cubic foot, then 1 cu. ft. $= \frac{1728}{2150.4}$ bu. $=$ *about* .8 bu.

Hence, the

RULE.—*Multiply the number of cu. ft. by* .8.

How many bushels will a bin hold:

2. Which is 6 ft. long, 5 ft. wide, and $2\frac{1}{2}$ ft. high?

Ans. 60 bu.

3. Which is 6 ft. long, $4\frac{1}{2}$ ft. wide, and $2\frac{2}{3}$ ft deep?

Ans. $57\frac{3}{5}$ bu.

4. Which is $5\frac{1}{3}$ ft. long, $3\frac{3}{4}$ ft. wide, and $2\frac{1}{2}$ ft. deep?

Ans. 40 bu.

5. Which is $7\frac{1}{5}$ ft. long, $2\frac{7}{9}$ feet wide, and $3\frac{1}{3}$ ft. deep?

Ans. $53\frac{1}{3}$ bu.

291. CASE VI.—To find the number of board feet in boards or planks.

A Board Foot is used in measuring boards, planks, and sawed timber generally. It is 1 foot long, 1 foot wide, and 1 inch thick, and contains 144 cu. in.

1. How many board feet in a plank 13 ft. long, 18 in. wide, and 2 in. thick? *Ans.* 39 B. ft.

OPERATION.

$$\frac{13 \times 18 \times 2}{12} = 39.$$

Hence, the

RULE.—*Multiply the length in feet by the width and thickness expressed in inches, and divide the product by* 12.

NOTE.—If a board or plank is less than 1 in. thick, it is disregarded; that is, the calculation is made as if it were 1 in. thick.

2. What is the number of feet in a board 15 ft. long, 9 in. wide, and $\frac{1}{2}$ in. thick. *Ans.* $11\frac{1}{4}$ B. ft.

3. How many feet in 12 planks, each 7 ft. long, 8 in. wide, and $1\frac{1}{2}$ in. thick? *Ans.* 84 B. ft.

4. How many feet in 9 boards, each 11 ft. long, 6 in. wide, and $\frac{3}{4}$ in. thick? *Ans.* $49\frac{1}{2}$ B. ft.

5. How many feet in 36 plank, each 10 ft. long, 11 in. wide, and $1\frac{1}{3}$ in. thick? *Ans.* 440 B. ft.

292. PARALLEL PROBLEMS.

1.[m] Albert was born Feb. 17, 1868, and Robert Feb. 17, 1875; how much older is Albert than Robert?

2. The Brooklyn Suspension Bridge was commenced Jan. 3, 1870, and was opened for travel July 4, 1882; how long was it in building?

Ans. 12 yr. 6 m. 1 da.

3.[m] How many acres in a field 8 ch. long and 5 ch. wide?

4. How many acres in a field 45 rd. long, and 38 rd. wide? *Ans.* 10 A. 2 R. 30 P.

5.m If carpeting is 4 ft. wide, how many yards will it take to cover a stage 12 ft. wide, and $5\frac{1}{3}$ ft. deep?

6. If carpeting is 5 ft. 6 in. wide, how many yards will be required to cover a floor $16\frac{1}{2}$ ft. long, and $15\frac{1}{3}$ ft. wide? *Ans.* $15\frac{1}{3}$ yd.

7.m How many marble slabs, 3 ft. long and 2 ft. wide, are necessary to pave a walk 150 ft. long, and 4 ft. wide?

8. How many bricks, 8 in. long and 4 in. wide, are required to construct a pavement 285 ft. long, and 7 ft. 4 in wide? *Ans.* 9405.

9.m What will it cost to cut a ditch 60 ft. long, 3 ft. wide, and 2 ft. deep, at $\$\frac{1}{2}$ per cu. yd?

10. A street is 600 ft. long, and 55 ft. wide; what will be the cost of elevating it 3 ft., at 60c. per cu. yd.? *Ans.* $2200.

11.m A tin box 3 ft. long, 2 ft. wide, and 1 ft. high, is full of honey; what is the honey worth at 50c. per gallon?

12. A cistern 6 ft. square at the bottom, and 10 ft. high, is 2 *thirds* full of water; how long will it supply a family that uses 45 gallons of water per day?

13.m A man sells corn at $\$\frac{1}{2}$ per bushel; what should be his price for a box of corn 5 ft long, 3 ft. wide, and 2 ft. deep?

14. Corn weighs 56 pounds per bushel; what is the weight of the corn in a granary 9 ft. long, 5 ft. wide, and 6 ft. high, the granary being 3 *fourths* full?

15.m If lumber is worth $\$1\frac{1}{2}$ per hundred board feet, what is the value of 20 planks, each 10 ft. long, 6 in. wide, and 1 in. thick?

16. If lumber is worth $18.50 per thousand board feet, what is the value of 36 plank, each 16 ft. long, 8 in. wide, $\frac{1}{2}$ in. thick? *Ans.* $7.104.

PERCENTAGE.

293. The term *per cent* means by the *hundred.*

Thus: 3 per cent means 3 hundredths, or $\frac{3}{100}$, or .03; 5 per cent means 5 hundredths, or $\frac{5}{100}$, or .05; 13 per cent means 13 hundredths, or $\frac{13}{100}$, or .13.

294. The process of calculating by *hundredths* is called Percentage.

INDUCTIVE EXERCISES.

295. 1. A man had 100 sheep, but lost 7 of them; what part of his sheep did he lose?

Ans. $\frac{7}{100}$, or 7 per cent.

2. A man had 50 sheep, but lost 3 of them; what part of his sheep did he lose?

Ans. $\frac{3}{50} = \frac{6}{100}$, or 6 per cent.

3. A man had 25 hogs, but lost 7 of them; what part of his hogs did he lose?

Ans. $\frac{7}{25} = \frac{28}{100} = 28$ per cent.

4. A man had \$200, and gained \$13 more; what part of his money was his gain?

Ans. $\frac{13}{200} = \frac{6\frac{1}{2}}{100} = 6\frac{1}{2}$ per cent.

5. What per cent of a number is $\frac{1}{2}$ of it?

Ans. $\frac{1}{2} = \frac{50}{100} = 50$ per cent.

6. What per cent of a number is:

$\frac{1}{3}$ of it? *Ans.* $33\frac{1}{3}$.	$\frac{1}{6}$ of it?	$\frac{1}{9}$ of it?	$\frac{1}{20}$ of it?
$\frac{1}{4}$ of it? *Ans.* 25.	$\frac{1}{7}$ of it?	$\frac{1}{10}$ of it?	$\frac{1}{25}$ of it?
$\frac{1}{5}$ of it? *Ans.* 20.	$\frac{1}{8}$ of it?	$\frac{1}{12}$ of it?	$\frac{1}{50}$ of it?

What fractional part of a number is 20 per cent of it? *Ans.* $\frac{20}{100} = \frac{1}{5}$.

7. What fractional part of a number is:

25 per cent of it?	*Ans.* $\frac{1}{4}$.	$33\frac{1}{3}$ per cent of it?	
50 per cent of it?	*Ans.* $\frac{1}{2}$.	100 per cent of it?	
75 per cent of it?	*Ans.* $\frac{3}{4}$.	$12\frac{1}{2}$ per cent of it?	

Definitions.

296. Rate is a given allowance.

297. When rate is expressed by the hundred, the number of hundredths taken or allowed is the **Rate Per Cent.**

298. Percentage is the result obtained by taking a certain per cent of any given number.

299. The Base is the number on which the percentage is computed.

Per cent is indicated by %. Thus, 6% is read: 6 per cent.

300. Case I.—The Base and Rate % given to find the Percentage.

Exercises.

301. 1. What is 6% of 150 sheep?

Analysis.—6% of 150 sheep is $\frac{6}{100}$ of 150 sheep, which is 9 sheep.

Hence, the

Rule.—*Multiply the base by the rate expressed decimally.*

How much is:

2. 2% of 450 bu.?	*Ans.* 9 bu.	5. 20% of \$185.70?	*Ans.* ?
3. 15% of 240 c.?	*Ans.* 36 c.	6. 50% of \$124.37?	*Ans.* ?
4. $8\frac{1}{2}$% of \$120?	*Ans.* \$10.20.	7. $33\frac{1}{3}$% of 183 lb.?	*Ans.* ?

How many dollars are made by selling:

8. A cow which cost \$18, so as to gain 20%? *Ans.* \$3.60.

9. A horse which cost \$135, so as to gain 15%? *Ans.* \$20.25.

10. A wagon which cost \$84, at an advance of $12\frac{1}{2}\%$? *Ans.* $\$10\frac{1}{2}$.

How many dollars are lost by selling:

11. A lot which cost \$2345, so as to lose 9%? *Ans.* \$211.05.

12. A carriage which cost \$245, so as to lose 8%? *Ans.* \$19.60.

13. A saddle which cost \$14, at a discount of $7\frac{1}{2}\%$? *Ans.* ?

302. CASE II.—The Base and Percentage given to find the Rate %.

EXERCISES.

303. 1. What per cent of 16 is 4?

ANALYSIS.—4 is $\frac{4}{16}$ of 16. $\frac{4}{16} = \frac{1}{4} = \frac{25}{100} = 25$ per cent.

Hence, the

RULE.—*Multiply the number which is the percentage by* 100, *and divide the product by the base.*

What per cent of:

2. 500 is 40? *Ans.* 8%.
3. 825 is 50? *Ans.* $6\frac{2}{3}\%$.
4. \$637 is \$38.22? *Ans.* 6%.
5. \$300 is \$37.50? *Ans.* $12\frac{1}{2}\%$.
6. 250 bu. is 18 bu.? *Ans.* $7\frac{1}{5}\%$.
7. £ 40 is £ 2 16 s.? *Ans.* 7%.
8. 50 gal. is 2 gal. 3 qt.? *Ans.* ?
9. 20 yd. is 5 ft. 10 in.? *Ans.* ?

10. A man bought a cow for \$40, and sold her so as to gain \$10; what per cent did he make? *Ans.* 25%.

11. A man bought a cow for \$50, and sold her at a loss of \$10; what per cent did he lose? *Ans.* 20%.

COMMISSION.

304. Commission is a sum of money paid to an agent for buying or selling goods or other property. It is a *percentage* of the amount *invested* or *collected.*

MENTAL EXERCISES.

305. How much does an agent get for his services who collects:

1. \$500, and charges 2 per cent? *Ans.* \$10.
2. \$650, and charges 4 per cent? *Ans.* ?
3. \$200, and charges $2\frac{1}{2}$ per cent? *Ans.* ?
4. \$325, and charges 8 per cent? *Ans.* ?

How much does an agent receive who sells goods to the amount of:

5. \$450, at 2% commission? *Ans.* \$9.
6. \$125, at 8% commission? *Ans.* ?
7. \$50, at $3\frac{1}{2}$% commission? *Ans.* ?

WRITTEN EXERCISES.

306. 1. A commission merchant sold wheat for \$1900; what was his commission at $2\frac{3}{4}$ per cent? *Ans.* \$52.25.

2. A commission merchant invested \$1250 for another party; what was his commision at $1\frac{3}{5}$ per cent? *Ans.* \$20.

What does a commission merchant receive for his services who sells:

3. 5 horses at \$150 each, at $2\frac{1}{2}$ per cent commission? *Ans.* \$18.75.

4. 12 bales of cotton, averaging 450 pounds to the bale, at 8 cents a pound, commission being $1\frac{2}{3}$ per cent? *Ans.* $7.20.

5. 6 hhds. of sugar, averaging 1450 pounds to the hhd., at 5 cents a pound, commission being $1\frac{3}{4}$ per cent? *Ans.* $7.61\frac{1}{4}$.

6. 120 bu. apples @ $1.50, 620 lb. butter @ $.25, 1250 lb. bacon @ 14 c., commission being $1\frac{3}{4}\%$? *Ans.* $8.92\frac{1}{2}$.

Profit and Loss.

307. Profit and **Loss** denote the gain or loss in business transactions. They are computed by percentage.

308. The cost is the base; the **per cent** of gain or loss, the **rate**; the **gain** or **loss**, the **percentage**. In case of a gain, the **selling price** is the **amount**, and in case of a loss, the **selling price** is the **difference.**

309. Parallel Problems.

1. A man bought a gun for $8 and sold it for $10; what per cent did he make? *Ans.* 25%.

Note.—The gain was $10 − $8 = $2, and the cost $8, and by Art. **303**, $2 are 25% of $8.

What per cent does a man make who buys:

2.m A saddle for $10, and sells it for $12? *Ans.* 20%.
3. A book for $42, and sells it for $63? *Ans.* 50%.
4.m A table for $3, and sells it for $6? *Ans.* ?
5. A cow for $120, and sells her for $220? *Ans.* $83\frac{1}{3}\%$.
6.m A horse for $60, and sells him for $90? *Ans.* ?

7. A man bought a coat for \$10, and sold it for \$8; what per cent did he lose? *Ans.* 20%.

NOTE.—The loss was \$10 − \$8 = \$2, and the cost \$10, and by Art. **303**, \$2 are 20% of \$10.

What per cent does a man lose who buys:

8.m A saddle for \$12, and sells it for \$10? *Ans.* $16\frac{2}{3}\%$.

9. A secretary for \$62, and sells it for \$40? *Ans.* $35\frac{15}{31}\%$.

10.m A table for \$6, and sells it for \$3? *Ans.* 50%.

11. A cow for \$25, and sells her for \$23? *Ans.* 8%.

12.m A horse for \$80, and sells him for \$60? *Ans.* ?

13. A horse for \$80.50, and sells him for \$62? *Ans.* ?

14. A farm for \$320, and sells it for \$280? *Ans.* $12\frac{1}{2}\%$.

What will a merchant receive for an article which:

15.m Cost \$10, if sold so as to gain 20%? *Ans.* \$12.

16. Cost \$122, if sold at a profit of 25%? *Ans.* \$152.50.

17.m Cost \$24, if sold at a gain of 5%? *Ans.* ?

18. Cost \$282, if sold at an advance of 15%? *Ans.* ?

19.m Cost \$8, if sold so as to lose 50%? *Ans.* \$4.

20. Cost \$53, if sold at a loss of 22%? *Ans.* \$41.34.

21.m Cost \$25, if sold at a loss of 12%? *Ans.* ?

22. Cost \$182, if sold at a discount of $12\frac{1}{2}\%$? *Ans.* ?

310. QUESTIONS FOR REVIEW.

What is: 1. Rate? 2. Rate per cent? 3. Percentage? 4. The Base?

How is per cent indicated?

How do we find: 1. The percentage, when the base and rate % are given? 2. The rate %, when the base and percentage are given?

What is: 1. Commission? 2. Profit and Loss?

INTEREST.

INDUCTIVE EXERCISES.

311. Do men pay for the *use* of horses borrowed from a livery stable? What is the money so paid called? *Ans.* **Hire.** How is *hire* estimated? *Ans.* On 1 horse for 1 day. If 1 horse for 1 day cost $2, what do we call the $2? *Ans.* The rate of hire. If the rate of hire is $2, what will be the hire of 2 horses for 3 days? 4 horses for 5 days? 6 horses for $3\frac{1}{2}$ days?

Do men pay for the *use* of borrowed land? What is the money so paid called? *Ans.* **Rent.** How is rent estimated? *Ans.* On 1 acre for 1 year. If 1 acre for 1 year cost $2, what do we call the $2? *Ans.* The rate of rent. If the rate of rent is $4, what will be the rent of 2 acres for 1 year? 40 acres for 1 year? 40 acres for 2 years? 80 acres for $1\frac{1}{2}$ years? 50 acres for 2 years 6 months, ($2\frac{1}{2}$ years)?

Do men pay for the *use* of borrowed money? What is the money so paid called? *Ans.* **Interest.** How is interest estimated? *Ans.* On 1 dollar for 1 year? If $1 for 1 year cost 6 c., what do we call the 6 c.? *Ans.* The rate of interest. If the rate of interest is 6 c., what will be the interest of $20 for 1 year? $20 for 2 years? $40 for 2 years?

What part of $1 is 6 c.? *Ans.* .06, or 6 per cent. Instead of saying "at the rate of 6 c. on $1," would it be the same to say: "at the rate of 6 per cent?" What does "at the rate of 8 per cent" mean? *Ans.* "At the rate of 8 c. on $1."

Definitions.

312. Interest is the sum paid for the use of money.

313. Principal is the money for the use of which interest is paid.

314. Amount is the sum of principal and interest.

Rate per cent is the number of cents allowed for the use of $1 for 1 year.

315. Case I.—When the time is expressed in years.

1. What is the interest of $40 for 2 years at 6 per cent?

Analysis.—Since the rate is 6%, the interest of $1 for 1 year is 6 c. Hence, the interest of $40 for 1 year will be 40 times 6 c. = 240 c., or $2.40, and for 2 years 2 times $2.40 = $4.80.

Mental Exercises.

316. What is the interest, at 6 per cent, of:

2.	$3 for 1 yr.? *Ans.* 18 c.	6.	$4 for $2\frac{1}{2}$ yr.? *Ans.* 60 c.
3.	$3 for 2 yr.? *Ans.* 36 c.	7.	$5 for $3\frac{1}{3}$ yr.? *Ans.* $1.
4.	$5 for 2 yr.? *Ans* ?	8.	$10 for $1\frac{1}{4}$ yr.? *Ans.* ?
5.	$10 for 3 yr?. *Ans.* $1.80.	9.	$15 for $\frac{1}{3}$ yr.? *Ans.* ?

Hence, the

Rule.—*Multiply the principal by the rate per cent, and the product will be the interest for* 1 *year; then multiply this product by the number of years.*

To find the Amount, add the Principal to the Interest.

Written Exercises.

317. What is the interest of:

1. \$120 for 3 yr. at 6%? At 8%? *Ans.* \$21.60; \$28.80.
2. \$120.40 for 4 yr. at 5%? At 7%?
Ans. \$24.08; \$33.712.
3. \$325 for 2 yr. at 9%? At 3%? *Ans.* \$58.50; \$19.50.
4. \$234.26 for 5 yr. at 6%? At 10%?
Ans. \$70.278; \$117.13.
5. \$180.04 for $2\frac{1}{2}$ yr. at 8%? At 7%?
Ans. \$36.008; \$31.507.
6. \$144 for $1\frac{1}{3}$ yr. at 6%? At 8%? *Ans.* ? ; ?

What is the amount of:

7. \$49 for 2 yr. at 5%? At 6%? *Ans.* 53.90; \$54.88.
8. \$325 for 3 yr. at 4%? At 5%? *Ans.* \$364; \$373.75.
9. \$63.75 for $3\frac{1}{2}$ yr. at 8%? At 10% *Ans.* ? ; ?
10. What is the interest of \$275 at 6% from June 12, 1881, to June 12, 1884? At 8%? *Ans.* \$49.50; \$66.

318. Case II.—When the time is expressed in years and months.

In this case we reduce the time to years.

What part of 1 year is:

1. 7 months? *Ans.* $\frac{7}{12}$.
2. 5 months? *Ans.* $\frac{5}{12}$.
3. 11 months? *Ans.* ?
4. 1 month? *Ans.* ?
5. 6 months? *Ans.* $\frac{1}{2}$.
6. 8 months? *Ans.* $\frac{2}{3}$.
7. 10 months? *Ans.* $\frac{10}{12} = ?$
8. 2 months? *Ans.* $= \frac{1}{6}$.
9. 3 months? *Ans.* $\frac{1}{4}$.
10. 9 months? *Ans.* $= ?$
11. 4 months? *Ans.* $= ?$
12. 12 months? *Ans.* $= ?$

How many years in:

13. 4 yr. and 3 mo.? *Ans.* $4\frac{1}{4}$.
14. 3 yr. and 8 mo.? *Ans.* $3\frac{2}{3}$.
15. 5 yr. and 6 mo.? *Ans.* ?
16. 2 yr. and 11 mo.? *Ans.* ?

Written Exercises.

319. 1. What is the interest of \$130.16 at 6% for 2 yr. 9 mo.?

Explanation.— We multiply the interest for 1 year by the time (2 yr. 9 mo.) reduced to yr. ($2\frac{3}{4}$).

Hence, the

Rule.—*Reduce the time to years and proceed as in Case I.*

OPERATION.

130.16	Principal.
.06	Rate %.
7.8096	Int. for 1 year.
$2\frac{3}{4}$	No. yr.
4)23.4288	Product by 3.
5.8572	Quotient by 4.
15.6192	Product by 2.
21.4764	Interest.

What is the interest and amount of:

2. \$936 for 2 yr. 5 mo. at 7%?

Ans. Int. \$158.34; Am't \$1094.34.

3. \$1248 for 3 yr. 8 mo. at 5%?

Ans. Int. ? Am't \$1476.80.

4. \$672.84 for 4 yr. 7 mo. at 8%?

Ans. Int. \$246.708; Am't ?

5. \$576.48 for 3 yr. 5 mo. at 6%?

Ans. Int. ? Am't \$694.6584.

6. \$120.60 for 1 yr. 1 mo. at 9%? *Ans.* Int. ? Am't?

7. \$864.18 for 3 yr. 10 mo. at 8%?

Ans. Int. ? Am't \$1129.1952.

8. \$960.48 for 9 mo. at 5%?

Ans. Int. \$36.018; Am't \$996.498.

9. What is the interest of \$320 from May 12, 1878, to July 12, 1881, at 6%? *Ans.* \$60.80.

10. What is the amount of \$20.15 from Dec. 17, 1880, to March 17, 1885, at 8%? *Ans.* \$27.001.

11. A man borrowed \$180.60 June 7, 1881, and settled the account Oct. 7, 1884; how much did he owe, the rate of interest being 8%? *Ans.* \$228.76.

320. CASE III.—When the time is expressed in years, months, and days.

In this case we reduce the time to months, thus:

1°. Since 12 months make 1 year, we multiply the number of years by 12, and to the product add the number of months.

Thus: 1 yr. 3 mo. = 15 mo.; 3 yr. 5 mo. = 41 mo.; etc.

2°. Since 30 days are reckoned as a month, 3 days is $\frac{1}{10}$, or .1 of a month; hence, we divide the number of days by 3, which reduces them to *tenths* of a month.

Thus: 15 da. = .5 mo.; 27 da. = .9 mo.; 11 da. = .3⅔ mo.; etc.

In this manner reduce to months:

1.	2 yr. 3 mo. 6 da.	*Ans.* 27.2 mo.
2.	1 yr. 7 mo. 21 da.	*Ans.* 19.7 mo.
3.	4 yr. 9 mo. 3 da.	*Ans.* 57.1 mo.
4.	5 yr. 2 mo. 19 da.	*Ans.* 62.6⅓ mo.
5.	1 yr. 1 mo. 1 da.	*Ans.* 13.0⅓ mo.
6.	5 yr. 28 da.	*Ans.* 60.9⅓ mo.
7.	7 mo. 14. da.	*Ans.* 7.4⅔ mo.
8.	3 mo. 22 da.	*Ans.* 3.7⅓ mo.
9.	26 da.	*Ans.* .8⅔ mo.
10.	95 da.	*Ans.* 3.1⅔ mo.

321. 1. What is the interest of $180.60 for 2 yr. 3 mo. 6 da. at 6%?

EXPLANATION.—Since the interest of any sum for one year at 6% is .06 of the principal, $180.60 × .06 gives the interest for 1 yr. or 12 mo. This product, divided by 12, gives the interest for 1 mo., which, multiplied by 27.2, the given time expressed in months, gives $24.5616, the required interest.

OPERATION.

$180.60	Principal.
.06	Rate %.
12)$10.8360	Int. for 1 yr.
.9030	Int. for 1 mo.
27.2	No. of mo.
$24.56160	Required Int.

EXPLANATION.—Draw a vertical line, on the right place the principal, the rate, and number of months, and on the left, 12. Now cancel common factors, and multiply.

Dividing 180.60 by 12, we get 15.05, which, multiplied by .06 and then by 27.2, gives $24.5616.

2d. OPERATION.

12	180.60 15.05
	.06
	27.2
	24.56160

Hence, the

RULE.—*Multiply the principal by the rate per cent, divide the product by* 12, *and multiply the quotient by the time expressed in months.*

Or, *Draw a vertical line, on the right place the principal, rate, and number of months; on the left,* 12; *cancel common factors on opposite sides of the line, and divide the product of the remaining factors on the right by the remaining term, if any, on the left.*

WRITTEN EXERCISES.

322. 2. What is the interest of $620 for 1 yr. 8 mo. 12 da. at 6%? *Ans.* ?

3. What is the interest of $840.60 for 3 yr. 5 mo. 6 da. at 7%? *Ans.* $202.0242.

4. What is the amount of $375 for 2 yr. 6 mo. 21 da. at 8%? *Ans.* ?

5. What is the amount of $656.84 for 4 yr. 10 mo. 15 da. at 6%? *Ans.* $848.9657.

What is the interest of:

6. $184.80 for 1 yr. 1 mo. 10 da. at 9%? *Ans.* $18.48.

7. $321.70 for 4 yr. 3 mo. 27 da. at 4%? *Ans.* $55.654.

8. $208.44 for 7 yr. 8 mo. 15 da. at 5%? *Ans.* $80.336.

9. $1365.40 for 11 mo. 27 da. at 6%? *Ans.* ?
10. $260.20 for 2 yr. 24 da. at 10%? *Ans.* ?
11. $240.60 for 1 yr. 2 mo. 19 da. at 5%? *Ans.* ?
12. $360.48 for 5 mo. 17 da. at 7%? *Ans.* ?

What is the amount of:

13. $1020.96 for 3 yr. 7 mo. 18 da. at 5%?
Ans. $1206.4344.
14. $672.24 for 2 yr. 2 mo. 25 da. at 6%? *Ans.* $762.4322.
15. $145.20 for 1 yr. 9 mo. 27 da. at $12\frac{1}{2}$%?
Ans. $178.323+.
16. $2500 for 7 mo. 20 da. at 5%? *Ans.* $2579.86.
17. $100.25 for 63 da. at 7%? *Ans.* $101.478+.
18. What is the interest of $1630 from April 1, 1878, to Oct. 10, 1882, at 6%? *Ans.* $442.545.
19. If $2150 are placed at interest May 10, 1877, what amount will be due Jan. 1, 1881, at 6%? *Ans.* $2619.775.
20. What is the interest of $540.36 from April 25, 1869, to December 15, 1872, at 6%? *Ans.* $117.9786.
21. What is the amount of $360.48 from Aug. 12, 1869, to March 27, 1872, at 8%? *Ans.* $436.1808.
22. A man borrowed $480.20 July 13, 1869, and kept it until Feb. 4, 1873; how much did he then owe at 6%?
Ans. ?
23. How much will an account of $175.50 amount to if contracted Nov. 17, 1875, and settled May 5, 1885, at 8% interest? *Ans.* ?

323. CASE IV.—When the time is expressed in days.

EXERCISES.

1. What is the interest of $450 at 8% for 91 days?
We may reduce the days to months and proceed as

in the last case; but the following method of solution is generally preferable:

EXPLANATION.—Multiplying \$450 by .08 gets the int. for 1 yr. or 360 da.; dividing this product by 360 gets the int. for 1 da., which, multiplied by 91, gives the required interest.

OPERATION.

$$\begin{array}{r|ll} \not{360} & \not{450} & 5 \\ \not{4} & .\not{08} & .02 \\ & 91 & \end{array}$$

Hence, the *Ans.* \$9.10.

RULE.—*Draw a vertical line, on the right of it place the principal, rate, and days; on the left,* 360, *and proceed according to the rule of cancellation.*

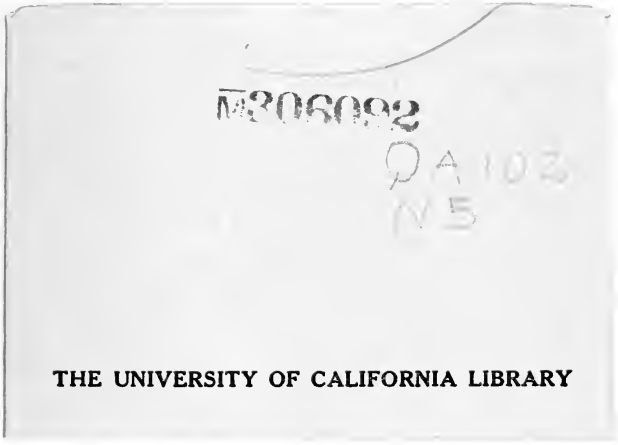
What is the interest of:

2. \$450 for 63 da. at 6%? *Ans.* \$4.725.
3. \$278.68 for 36 da. at 10%? *Ans.* \$2.7868.
4. \$600 for 93 da. at 5%? *Ans.* \$7.75.
5. \$720 for 125 da. at 7%? *Ans.* \$17.50.
6. \$480.60 for 148 da. at 9%? *Ans.* \$17.7822.
7. \$563.25 for 200 da. at 12%? *Ans.* \$37.55.

324. QUESTIONS FOR REVIEW.

What is: 1. Interest? 2. The Principal? 3. The rate per cent? 4. The amount?

What is the rule for finding interest when the time is expressod in: 1. Years? 2. Years and months? 3. Years, months, and days? 4. Days?



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